EFFECT OF RADIATION ON HEAT TRANSFER FROM A FLOWING FLUID IN A CIRCULAR PIPE

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EFFECT OF RADIATION ON HEAT TRANSFER FROM A FLOWING FLUID IN A CIRCULAR PIPE

by

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B. S. in M. E., CORNELL UNIVERSITY
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Cambridge, Massachusetts
May 23, 1955.

Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dear Sir:

In accordance with the requirements for the Degree of Naval Engineer, I hereby submit this thesis entitled "Effect of Radiation on Heat Transfer from a Flowing Fluid in a Circular Pipe."

Respectfully yours,

Thomas H. Hopkins
Lieutenant
U. S. Navy

28786

ABSTRACT

The influence of radiation heat transfer on the forced convection dominated heat transfer problem has not been established. An analytic solution for this problem is presented for the combined radiation and convection heat transfer from a turbulently flowing hot fluid in a long circular pipe. A sample numerical calculation is carried to completion for pure carbon dioxide gas flowing in a two inch diameter pipe. Constant mean value fluid properties are assumed throughout the analysis. The results of this restricted solution show that radiation effects do cause a gain in the total net heat transfer rate to the pipe wall compared with an equivalent zero radiation solution. However, it is also shown that the sum of independent radiation and convection solutions cannot be used for an adequate description of the combined radiation and convection heat transfer solution.

ACKNOWLEDGMENTS

The need for an analytic solution to this combined radiation and convection heat transfer problem was first brought to my attention by Professor Warren M. Rohsenow of the Mechanical Engineering Department of M.I.T. His suggestions and counsel led me to the general extension of the von Karman convection heat transfer analysis for pipe flow to include the effects of radiation.

Special thanks are due to Professor Hoyt C. Hottel of the Chemical Engineering Department at MIT for providing the method of solution for the total net radiation heat flux received by any infinitesimal control volume of gas in the pipe. His consultation on many of the other radiation considerations involved in this thesis was also of special value.

Edward S. Cohen, graduate student in Chemical Engineering at MIT, was of assistance on frequent occasions in giving me instruction and advice on many of the radiation aspects of this thesis.

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LIST OF SYMBOLS

Symbol	Definition	Units
A	Area normal to the axis of the pipe at any radius	$foot^2$
a	Dimensionless distance between two element control volumes projected into the $r - \theta$ plane	-----
c_p	Specific heat at constant pressure	Btu/lb$^\circ$
D	Pipe diameter	foot
E	Radiation-emission rate, $E = \sigma T^4$	Btu/hr.
f	Dimensionless friction factor	-----
g_o	Conversion factor, 32.174 (lb.mass)(foot)/(lb.force)(sec.^{2})	lb.ft/l
h	Coefficient of heat transfer between fluid and surface for zero radiation	Btu/hr.
i	Enthalpy of fluid	Btu/lb.
K_a	Absorption coefficient	(feet)$^-$
K_ϵ	Emission coefficient	(feet)$^-$
k	Thermal conductivity	Btu/hr.
L	Mean path length for radiation	feet
N_{Nu}	Nusselt number = (hd/k)	-----
N_{Pr}	Prandtl number = $(c_p \mu/k)$	-----
N_{Re}	Reynolds number = $(\rho VD/g_o \mu)$	-----
N_{St}	Stanton number = $(h/c_p \rho V)$	-----
P_G	Pressure of the gas	atmosph
p	Pressure of the gas	lb/ft^2
Q	Heat transfer to a given control volume	Btu
q	Heat transfer rate to a given control volume	Btu/hr.

Symbol	Definition	Units
R	Pipe radius ($R = r_o$)	feet
r	Radius to any given point	feet
T	Absolute temperature	°R
t	Temperature	°F
V	Average fluid velocity	feet/sec.
v	Local fluid velocity	feet/sec.
v^+	Dimensionless local fluid velocity $\left(v^+ = v \sqrt{\frac{\rho}{g_c \tau_o}} \right)$	-----
X	Dimensionless axial distance	-----
x	Axial distance to any given point	feet
$(x)_g$	Real gas transmittance weighting factor	-----
y	Distance from the wall ($y = r_o - r$)	feet
y^+	Dimensionless distance from the wall, $\left(y^+ = \frac{y}{\nu} \sqrt{\frac{g_c \tau_o}{\rho}} \right)$	-----
z	Distance in space between two element control volumes	feet
α	Ratio of eddy diffusivities $\alpha = \epsilon_h / \epsilon_m$	-----
α_g	Absorptivity of the gas for radiation	-----
β	Angle made by a radiant beam with normal to surface element.	-----
ϵ_h	Eddy diffuvisity for heat	ft^2/hr
ϵ_m	Eddy diffuvisity for momentum	ft^2/hr
ϵ	Emissivity	-----
θ	Angular position of any given point	-----

Symbol	Definition	Units
μ	Viscosity of the fluid	centipoise
ν	Kinematic viscosity	ft^2/hr
σ	Stefan-Boltzmann constant	$Btu/hr.ft^2(°R)^4$
τ	Shear stress at any point	lb/ft^2
ψ	Element control volume relative position angle	-----

Subscripts

a	absorption
b	bulk
e	emission
f	film
g	gas
h	heat
m	momentum
o	wall or gas at wall
r	radiation, or r direction
w	wall
x	x direction
θ	θ direction
℄	centerline

INTRODUCTION

Fluid flow heat transfer problems involving high temperatures have become more significant recently with the growing number of high temperature fluid applications.

At these higher temperatures, all fluid flow heat transfer problems involving the heteropolar gases may be significantly affected or controlled by thermal radiation. The two most noteworthy gases that fit into this category are water vapor and carbon dioxide.

The analytical solution for any heat transfer problem involving forced convection heat transfer in a turbulent flowing fluid, coupled with thermal radiation to and from that fluid, is relatively difficult to obtain.

For the simple case of steady flow of a fluid in a circular pipe, the independent forced convection solution has received good analytical treatment, especially when moderate temperature extremes will permit the assumption of constant fluid properties. [5] or [10]

The independent radiation problem involving a non-uniform temperature distribution in a gas has received very little analytical treatment. One method of solution [1] for this radiation problem has been presented, with a sample numerical solution for a furnace type problem in which fluid convection heat transfer is relatively small.

The coupling effect of combining the convection heat transfer with the radiation increases the complexity of the problem. The two independent solutions cannot be simply added

together to produce an adequate net heat transfer solution. In fact, some experimental evidence* has indicated that the radiation, coupled with the convection heat transfer from a radiating gas, is actually detrimental. For instance, the net heat transfer from a hot turbulent flowing radiating gas such as carbon dioxide would be less than the net heat transfer from a non-radiating gas such as nitrogen under identical conditions. The limited experimental evidence is not conclusive, but it is apparent that the radiation effects are significant in heat transfer from radiating gases at higher temperatures.

The objective of this thesis is to carry out an analytical study of the effect of radiation on the net heat transfer from a hot turbulent flowing radiant gas in a cool cylindrical tube. This analysis will be limited to moderate temperature extremes by the assumption of constant fluid properties.

---

* Personal communication with [illegible], Purdue University

PROCEDURE

The first step in this analysis required a thorough review and study of the independent forced convection heat transfer analysis for turbulent flow of fluids inside tubes.

This analysis was then repeated in detail with the addition of the necessary radiation terms. The method of this solution employed the analogy between radial transfer of heat and momentum in long tubes, as carried to solution in the von Karman analysis. [10]

The following boundary conditions and limitations were imposed by this solution:

1. Examination is restricted to a given cross-section of an axisymmetric fluid flow in a long circular tube.
2. Moderate temperature extremes exist so
   - (a) physical properties may be assumed to be independent of temperatures
   - (b) temperature gradients have no effect on the velocity gradients.
3. The fluid is in a state of "steady" fully developed turbulent flow. The velocity distribution is given by the experimental data of Nikuradse. [6] page 154

The details of the first phase (Phase A) of the analysis are shown in Appendix I. The steps of this analysis are outlined as follows:

PHASE A - EXTENSION OF THE [illegible] TO THERMAL RADIATION

Step 1

Select a differential element control volume at a given cross-section. Taking advantage of this axisymmetric case, the element selected at this stage is of dimensions $2\pi r$ by dr by dx as shown in Figures I and II.

Step 2

Carry out the momentum transfer analysis in order to obtain the expression for the apparent shear stress at any radius. The momentum equation may be simplified to the following*

$$\frac{\partial p}{\partial x} = \frac{1}{r}\frac{\partial r}{\partial r}\left[r\left(\mu \frac{\partial \overline{v_x}}{\partial r} - \frac{\rho}{g_o}\overline{v_r' v_x'}\right)\right] \qquad (1)$$

Define the eddy diffusivity of momentum, ε_m, such that

$$\left[\overline{v_r' v_x'} \equiv -\varepsilon_m \frac{\partial \overline{v_x}}{\partial r}\right] \qquad (2)$$

Then the apparent shear stress becomes

$$\tau_{app} \equiv \tau_{molec} + \tau_{eddy} = -\left(\mu + \frac{\rho}{g_o}\varepsilon_m\right)\frac{\partial \overline{v_x}}{\partial r} \qquad (3)$$

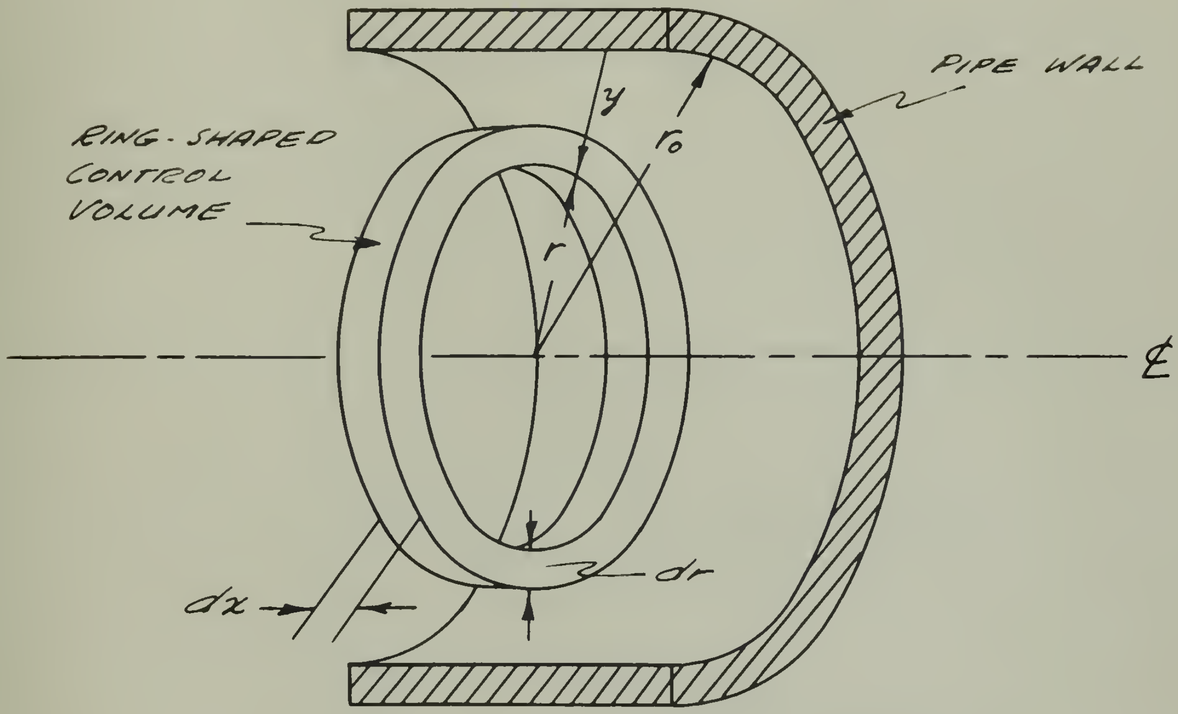
Also determine the distribution of this apparent shear stress with radius assuming $\left(\frac{\partial p}{\partial x}\right)$ is constant and independent of the radius at any given cross-section.

$$\left[\frac{\tau_{app}}{\tau_o} = \frac{r}{r_o} = \left(1 - \frac{y}{r_o}\right)\right] \qquad (4)$$

---

* See Appendix I A

Figure I



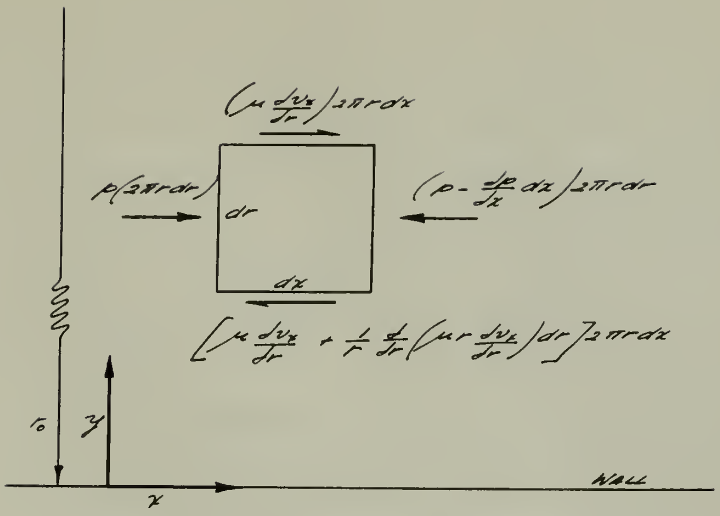

$(\mu \frac{dv_x}{dr})2\pi r\,dx$
$p(2\pi r\,dr)$
dr
$(p - \frac{dp}{dx}dx)2\pi r\,dr$
dx
$[\mu \frac{dv_x}{dr} + \frac{1}{r}\frac{d}{dr}(\mu r \frac{dv_x}{dr})dr]2\pi r\,dx$
r_0
y
x
WALL

Figure II a - Forces acting on fluid within control volume.

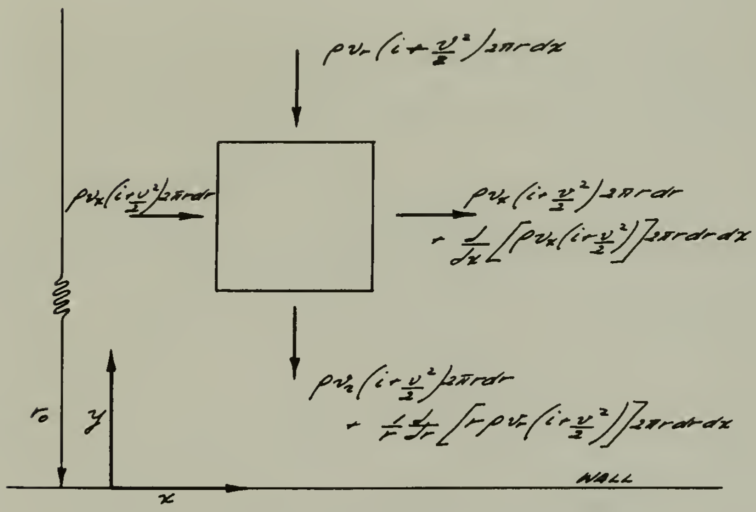

$\rho v_r(i + \frac{v^2}{2})2\pi r\,dx$
$\rho v_x(i + \frac{v^2}{2})2\pi r\,dr$
$\rho v_x(i + \frac{v^2}{2})2\pi r\,dr$
$+ \frac{d}{dx}[\rho v_x(i + \frac{v^2}{2})]2\pi r\,dr\,dx$
$\rho v_r(i + \frac{v^2}{2})2\pi r\,dr$
$+ \frac{1}{r}\frac{d}{dr}[r \rho v_r(i + \frac{v^2}{2})]2\pi r\,dr\,dx$
r_0
y
x
WALL

Figure II b - Fluxes of enthalpy and kinetic energy through the control surface.

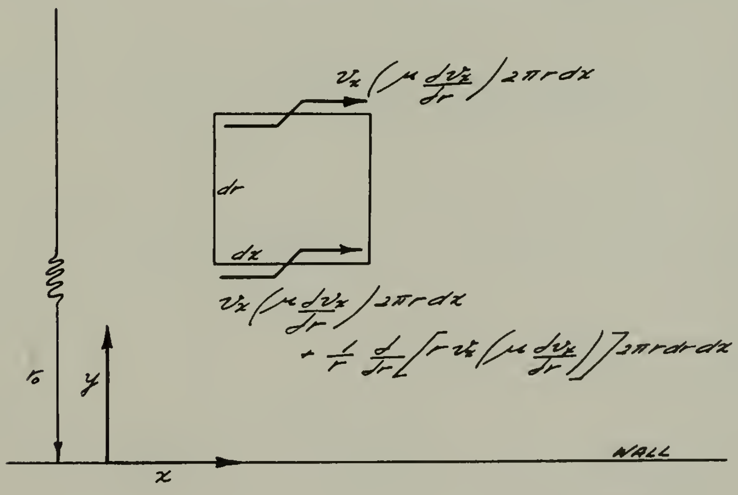

$v_x(\mu \frac{dv_x}{dr})2\pi r\,dx$
dr
dx
$v_x(\mu \frac{dv_x}{dr})2\pi r\,dx$
$+ \frac{1}{r}\frac{d}{dr}[r v_x(\mu \frac{dv_x}{dr})]2\pi r\,dr\,dx$
r_0
y
x
WALL

Figure II c - Rates of shearing work at the control surface

Step 3

Carry out an energy balance on this element. Introduce the continuity and momentum equations, then drop all second order terms and contributions of relatively minor significance to obtain the following energy equation as a result.*

$$\frac{\partial(\rho \bar{v}_x \bar{I})}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[r \left\{ k \frac{\partial T}{\partial r} - c_p \overline{v_r' T'} \right\} \right] + \left(\frac{dq}{dv} \right)_r \qquad (5)$$

where $\left(\frac{dq}{dv} \right)_r$ is the net radiation heat transfer rate to this control volume per unit volume of gas.

Step 4

Define the eddy heat transfer diffusivity, ε_h, such that $v_r' T' \equiv - \varepsilon_h \frac{\partial T}{\partial r}$ and define apparent heat transfer flux, $(q/A)_{app}$, (6) such that at any given radius.

$$(q/A)_{app} \equiv \underbrace{(q/A)_{molecular} + (q/A)_{eddy}}_{\text{(Forced convection)}} + \underbrace{(q/A)_r}_{\text{(Radiation)}} \qquad (7)$$

or

$$(q/A)_{app} = (k + \rho c_p \varepsilon_h) \frac{\partial T}{\partial r} + (q/A)_r \qquad (8)$$

where $(q/A)_r$ is a function representing the radiation heat transfer rate at this point per unit area normal to the axis. (This term is evaluated in Phase B)

$$\text{Then } \frac{\partial(\rho \bar{v}_x I)}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[r \left\{ (q/A)_{app} - (q/A)_r \right\} \right] + \left(\frac{dq}{dv} \right)_r \qquad (9a)$$

and if $\frac{1}{r}\frac{\partial}{\partial r}\left[r \, (q/A)_r \right] \approx \left(\frac{dq}{dv} \right)_r$ or if radiation is (

relatively small then

$$\frac{\partial(\rho \bar{v}_x I)}{\partial x} \approx \frac{1}{r}\frac{\partial}{\partial r}\left[r \, (q/A)_{app} \right] \qquad (9b)$$

---

* See Appendix I B

Step 5

Evaluate the distribution of the apparent heat transfer flux, $(q/A)_{app}$, with radius. The following assumptions are required in order to establish this relationship.

(a) The heat transfer coefficient $h = \dfrac{(q/A)_o}{T_o - T_b}$ (10)

is very nearly uniform along the pipe length. (Radiation heat transfer is assumed to represent a small fraction of the total heat transfer at the wall.)

(b) For a given problem at specified values of the Reynolds number, Prandtl number and the radiation properties of the fluid, there exists a generalised temperature distribution which is nearly uniform in the direction of flow. Therefore

$$\frac{\partial}{\partial x}\left(\frac{T_o - T}{T_o - T_b}\right) = 0 \qquad (11)$$

For the case of uniform wall heat transfer flux, $(q/A)_o$, along the length of the pipe; this reduces to

$$\frac{\partial T}{\partial x} = \frac{\partial T_o}{\partial x} = \frac{\partial T_b}{\partial x}, \text{ independent of } r. \qquad (11a)$$

(c) The time average value of the axial velocity component, $\overline{v_x}$, is assumed to be approximately constant and independent of radius, (slug flow).

Then for the case of uniform wall heat transfer flux along the pipe, the distribution of the apparent heat transfer flux with radius is simply linear.

$$\frac{(q/A)_{app}}{(q/A)_o} = \frac{r}{r_o} = (1 - y/r_o) \qquad (12)$$

Even if the axial velocity is variable, as in turbulent flow, this apparent heat flux is still reasonably correct within the limitations of the other basic assumptions."

Step 6

Introduce the analogy between friction and heat transfer. From the momentum equations:

$$\tau_{app} = -\left(\mu + \frac{\rho}{g_o}\varepsilon_m\right)\frac{\partial v_x}{\partial r} \qquad (3)$$

and

$$\frac{\tau_{app}}{\tau_o} = \frac{r}{r_o} = \left(1 - \frac{y}{r_o}\right) \qquad (4)$$

Therefore we may obtain

$$\left[\frac{g_o\tau_o}{\rho}\left(1 - \frac{y}{r_o}\right) = \left(\nu + \varepsilon_m\right)\frac{dv_x}{dy}\right] \qquad (13)$$

From the heat transfer equations:

$$(q/A)_{app} = +\left\{\left(k + \rho c_p\varepsilon_h\right)\frac{\partial t}{\partial r} + (q/A)_r\right\} \qquad (8)$$

---

* See Reference [illegible]

and

$$\frac{(q/A)_{app}}{(q/A)_o} = \frac{r}{r_o} = \left(1 - \frac{y}{r_o}\right) \qquad (12)$$

Therefore we may obtain

$$\frac{(q/A)_o}{\rho c_p}\left(1 - \frac{y}{r_o}\right) = -\left(\frac{\nu}{Pr} + \varepsilon_h\right)\frac{dt}{dy} + \frac{1}{\rho c_p}(q/A)_r \qquad (14)$$

The analogy between equations (13) and (14) is obvious, but unfortunately a full analogy is now marred by the additional radiation term in the heat transfer equation. This term will receive more detailed examination during Phase I.

Step 7

Carry out the von Karman method of analysis of the pair of differential equations (13) and (14) in order to combine them into a single differential equation describing the temperature distribution. The velocity distribution data of Nikuradse [7] (page 174) is used as a basis to relate the velocity (v) with the distance from the wall (y) in terms of their dimensionless equivalents (v^+) and (y^+).

$$v^+ = f(y^+) \qquad (15)$$

$$\frac{d(v^+)}{d(y^+)} = f'(y^+) \qquad (16)$$

The ratio of the eddy diffusivities is expressed by

$$\alpha = \frac{\varepsilon_h}{\varepsilon_m} \qquad (17)$$

These expressions are combined with equations (13) and (14) to obtain the following differential equation relating t and y^+

$$-dt = \frac{(q/A)_o\ (1-y/r_o) - (q/A)_r}{\rho c_p \alpha \sqrt{\frac{g_o \tau_o}{\rho}} \left[\frac{1}{\alpha N_{Pr}} + \frac{(1 - y^+/r^+_o) - 1}{f'(y^+)}\right]}\ dy^+ \qquad (18)$$

Step 8

Integrate this temperature distribution equation to find the temperature t as a function of y^+. The first half of this equation can be integrated directly after certain simplifications are made as shown in the von Karman solution. The second part involving the radiation heat flux will require graphic integration, and will be discussed in Phase [illegible].

Integration is carried out separately in each of the three zones which are used to describe the velocity distribution.

(a) Zone I, Laminar layer, $(0 < y^+ < 5)$

$$[v^+ = y^+] \quad , \quad [f'(y^+) = 1] \qquad (19)$$

assume

$$(1 - \frac{y^+}{r^+_o}) = 1$$

Then

$$(T_o - T) = \frac{(q/A)_o (\alpha N_{Pr})(-y^+)}{\rho c_p \alpha \sqrt{\frac{g_o \tau_o}{\rho}}} - \frac{(\alpha N_{Pr})}{\rho c_p \alpha \sqrt{\frac{g_o \tau_o}{\rho}}} \int_0^{y^+} (q/A)_r\, dy^+ \qquad (20)$$

(b) Zone II, Buffer layer, $(5 < y^+ < 30)$

$$v^+ = 3.05 + 5\ln y^+ \quad , \quad f'(y^+) = (5/y^+) \tag{21}$$

Assume $(1 - \frac{y^+}{r_o^+}) = 1$

Then

$$T_5 - T = \left\{ \begin{array}{l} \dfrac{(q/A)_o \left[5\ln\left(1 - \alpha N_{Pr} + \alpha N_{Pr}\dfrac{y^+}{5}\right)\right]}{\rho c_p \alpha\sqrt{\dfrac{g_o \tau_o}{\rho}}} \\ \quad - \dfrac{5}{\rho c_p \alpha\sqrt{\dfrac{g_o\tau_o}{\rho}}} \displaystyle\int_5^{y^+} \dfrac{(q/A)_r \, dy^+}{\left[\dfrac{5}{\alpha N_{Pr}} - 5 + y^+\right]} \end{array} \right\} \tag{22}$$

(c) Zone III, Turbulent core, $(30 < y^+ < r_o^+)$

$$v^+ = 5.5 + 2.5\ln y^+ \quad , \quad f'(y^+) = (2.5/y^+) \tag{23}$$

Assume $\nu \ll \varepsilon_m$

and $(\frac{\nu}{N_{Pr}} \ll \varepsilon_h)$

Then

$$T_{30} - T = \frac{(q/A)_o\left[2.5\ln\left(\frac{y^+}{30}\right)\right]}{\rho c_p \alpha\sqrt{\frac{g_o\tau_o}{\rho}}} - \frac{2.5}{\rho c_p \alpha\sqrt{\frac{g_o\tau_o}{\rho}}}\int_{30}^{y^+}\frac{(q/A)_r\, dy^+}{\left[y^+(1 - y^+/r_o^+)\right]} \tag{24}$$

This is as far as the analysis can be carried without evaluation of the radiation heat transfer function $(q/A)_r$.

PHASE [illegible] - EVALUATION OF THE RADIATION HEAT TRANSFER FUNCTION

In order to evaluate the radiation heat transfer crossing the boundaries of the given ring-shaped control volume, it is

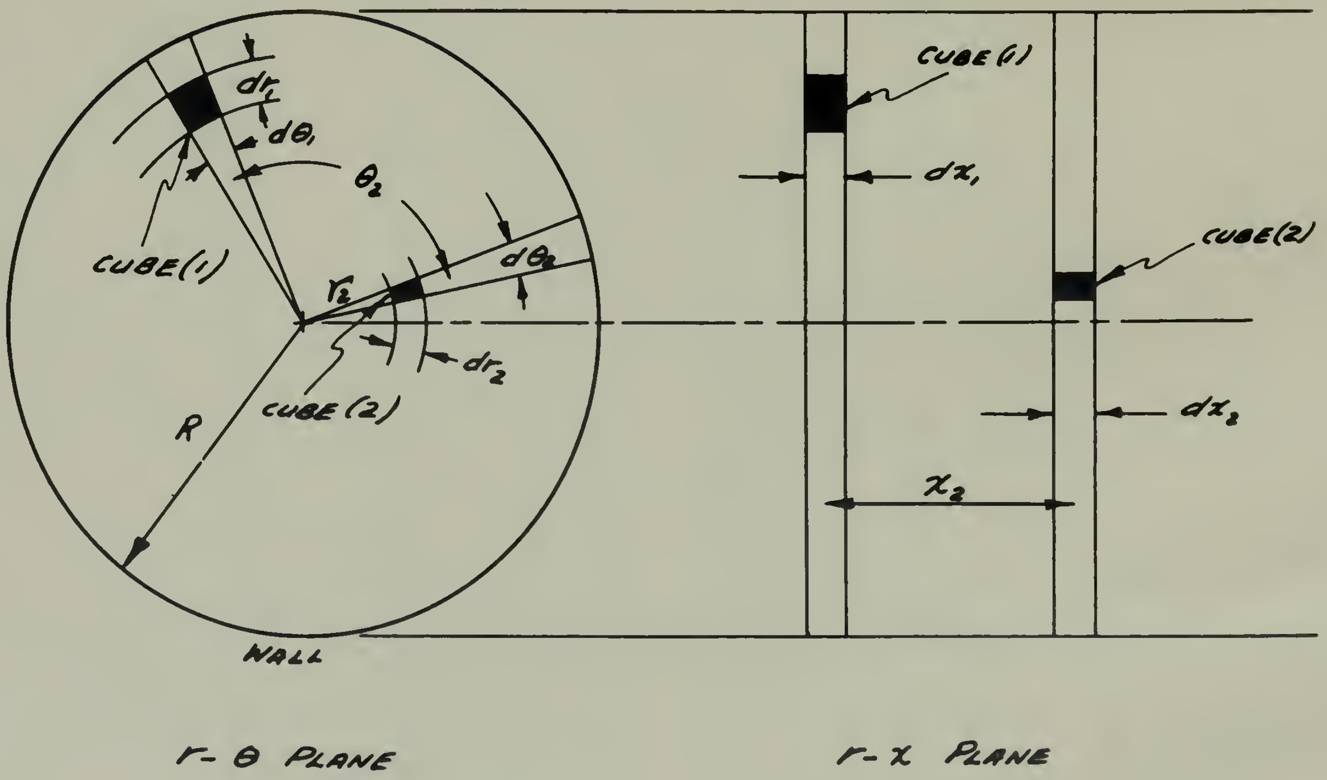

dr₁
dθ₁
θ₂
CUBE(1)
r₂
dθ₂
dr₂
CUBE (2)
R
WALL
r- θ PLANE
CUBE (1)
dx₁
CUBE(2)
dx₂
x₂
r- x PLANE

Figure III

convenient to focus attention on one representative small cube control volume in that ring. This cube is located at specified cylindrical coordinates r_1, θ_1 and x_1 and has dimensions dr_1 by $r_1 d\theta_1$ by dx_1. The net radiation heat transfer transmitted to this cube may be divided into three components: (1) emission from the cube, (2) absorption of radiation from the pipe walls and (3) absorption of radiation from all the other gas in the pipe.

Unfortunately, the nature of the radiation problem does not permit reduction to a relatively simple two-dimensional problem as in the case of the convection evaluation. The up and down stream contributions to the overall radiation heat transfer to a given cube are significant and necessary for an adequate evaluation.

First, the absorption of radiation from all the other gas in the pipe will be considered. The gas is assumed to be gray for the time being for simplicity. This will be corrected later on. The pipe wall is assumed to be black.

Step 1

Evaluate the radiation emission $(d^4q)_g$ which is received by the given cube (1) at r_1, θ_1 and x_1 from some other cube (2) at distance (z) located at r_2, θ_2 and x_2 as shown in Figure III. The description of the evaluation of this quantity is given in detail for the general case by [illegible] Cohen. [1] The one way radiation $(d^4q)_g$ from cube (2), (dv_2), to cube (1), (dv_1), is equal to the emission from cube (2), $(d^{4}e_{g}dv_{2})$,

multiplied by the transmittance of the intervening gas, ($e^{-K_a z}$), and by the absorptivity of cube (1),

$$\left(1 - e^{\frac{K_a dv_1}{4\pi z^2}}\right) = \left(\frac{K_a dv_1}{4\pi z^2}\right)$$ for an element of differential thickness .

So:

$$(d^4q)_g = \frac{(4 K_e E_2 dv_2)\ (K_a dv_1)\ e^{-K_a z}}{4\pi z^2} \tag{25a}$$

Then for this case:

$$(d^4 q)_g = \frac{(4 K_e E_2 r_2\, dr_2\, d\theta_2\, dx_2)(K_a r_1\, dr_1\, d\theta_1\, dx_1)\, e^{-K_a\sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta_2 + x_2^2}}}{4\pi\left(r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta_2 + x_2^2\right.} \tag{25b}$$

where $E_2 = \sigma(T_2)^4$. K_e is the emission coefficient for the gas and K_a is the absorption coefficient.* It is convenient to rewrite this equation in dimensionless form.

Let $a^2 = (r_1/R)^2 + (r_2/R)^2 - 2(r_1/R)(r_2/R)\cos\theta_2$,

$X = (x_2/R)$,

and $dv_1 = r_1 dr_1 d\theta_1 dx_1$

Then

$$(d^4 q)_g = \left[\frac{1}{\pi}(K_e R)(K_a R)(E_2 R^2)\left(\frac{r_2}{R}\right) d\left(\frac{r_1}{R}\right) d\theta_2 \left(\frac{dv_1}{R^3}\right) \frac{e^{-K_a R\sqrt{a^2 + X^2}}\, dX}{(a^2 + X^2)} \right. \tag{26}$$

---

* See Appendix II A

Step 2

Integrate this expression along the complete length of the pipe in order to obtain the radiation emission $(d^3q)_g$ which is received by cube (1) from a thin gas rod element located at r_2 and θ_2 extending the full length of the pipe.

At this point it is necessary to make some assumption concerning the axial temperature variation at r_2. It is assumed for this problem that the axial temperature gradient is relatively small (and constant with radius at any given cross-section as shown in Phase A). Therefore, at any given cross-section the temperature at any given radius will be assumed constant with length for purposes of this integration.

Therefore:

$$(d^3q)_g = \left[\frac{2}{\pi}(K_eR)(K_aR)(E_2R^2)\left(\frac{r_2}{R}\right)d\left(\frac{r_2}{R}\right)d\theta_2\left(\frac{dv_1}{R^3}\right)\right]\int_{X=0}^{X=\infty}\frac{e^{-K_aR\sqrt{a^2+X^2}}\,dX}{(a^2+X^2)} \tag{27}$$

It is more convenient for numerical evaluation if infinity is avoided, so

$$\text{let } \frac{a}{\sqrt{a^2+X^2}} = \cos\psi, \quad \sin\psi = \frac{X}{\sqrt{a^2+X^2}}$$

$$\text{and } d\psi = \frac{a\,dX}{a^2+X^2}$$

Then

$$(d^3q)_g = \left[\frac{2}{\pi}(K_eR)(K_aR)(E_2R^2)\left(\frac{r_2}{R}\right)d\left(\frac{r_2}{R}\right)d\theta_2\left(\frac{dv_1}{R^3}\right)\right]\left[\frac{1}{a}\int_{\psi=0}^{\psi=\pi/2}e^{-\frac{K_aRa}{\cos\psi}}\,d\psi\right] \tag{28}$$

This integral can be carried out numerically for a suitable range of values of (K_aRa). The plot of $(e^{-\frac{K_aRa}{\cos\psi}})$ versus at constant (K_aRa) value is shown in Figure IV. The integrals of these curves multiplied by $(\frac{1}{a})$ are plotted against (a) at constant values of (K_aR) in Figure V. These curves are then cross-plotted on semi-log paper, $Lg\left[\frac{1}{a}\int_0^{\pi/2} e^{-\frac{K_aRa}{\cos\psi}}\,d\psi\right]$ versus (K_aR) at constant (a) in order to aid in interpolation to any desired value of (K_aR).* It should be noted that these curves can be used for a wide range of pipe flow radiation problems and are not restricted to any specific numerical example.

Step 3

Determine the radiation emission $(d^2q)_g$ which is received by cube (1) from a thin cylindrical shell of gas extending the full length of the pipe symmetrically located about the center at radius r_2. The previous expression is then to be integrated about θ_2 at constant r_2.

$$(d^2_{\ b}q)_g = \left[\frac{4}{\pi}(K_eR)(K_aR)(E_2R^2)\left(\frac{r_2}{R}\right)d\left(\frac{r_2}{R}\right)\left(\frac{\psi_{2-1}}{R^3}\right)\right]\int_{\theta_2=0}^{\theta_2=\pi}\left[\frac{1}{a}\int_{\psi=0}^{\psi=\pi/2} e^{-\frac{K_aRa}{\cos\psi}}\,d\psi\right]d\theta_2 \quad (20)$$

Note that (a) is a known function of θ_2 for any given value of $(\frac{r_1}{R})$ and $(\frac{r_2}{R})$. It is therefore most convenient to carry out this integral at some specified value or series of values of the parameter K_aR.

---

* See Figures VI, VII, and VIII

Figure IV

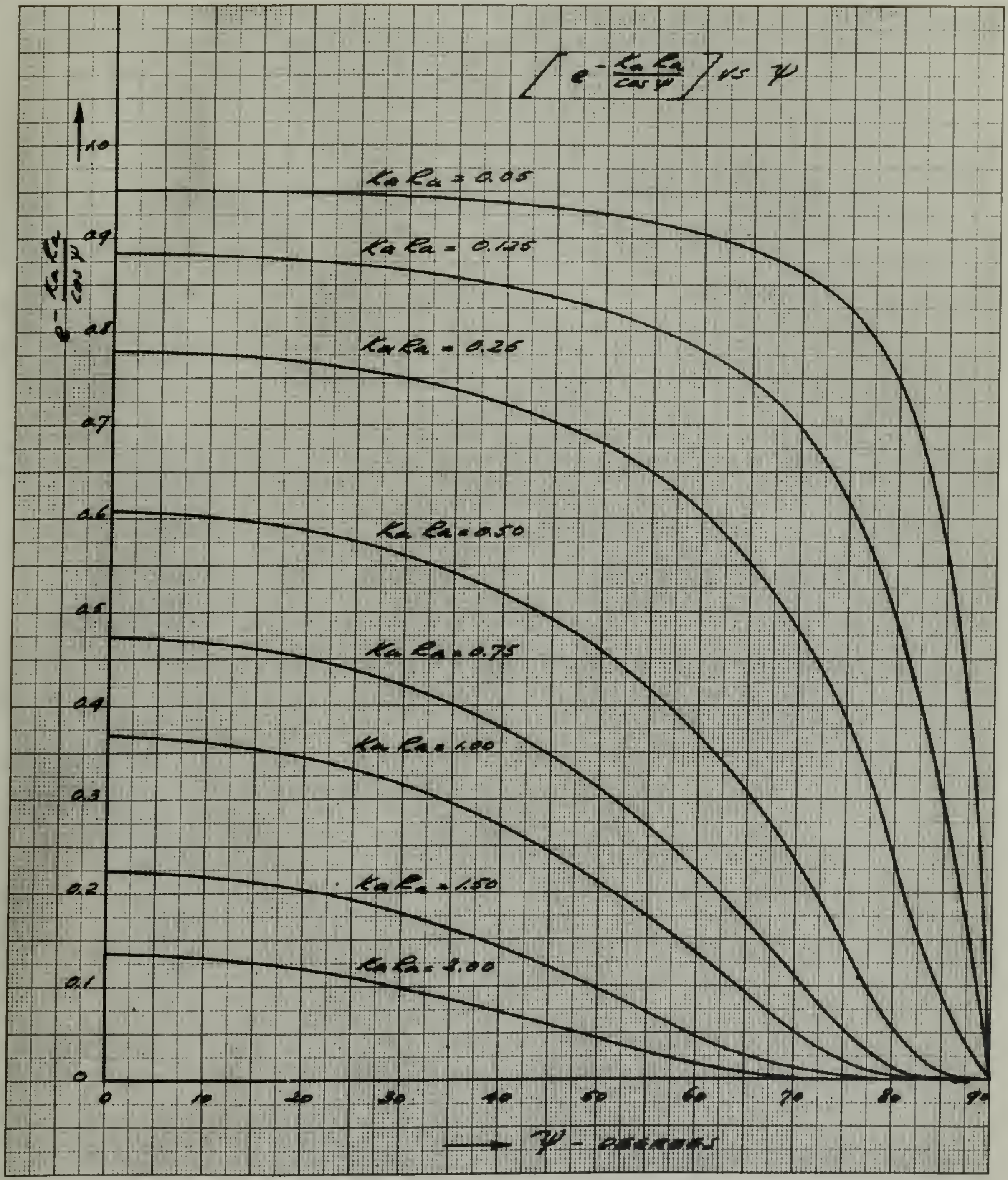

[e^- ka Ra / cos ψ] vs ψ
e^- ka Ra / cos ψ
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
ka Ra = 0.05
ka Ra = 0.125
ka Ra = 0.25
ka Ra = 0.50
ka Ra = 0.75
ka Ra = 1.00
ka Ra = 1.50
ka Ra = 2.00
0
10
20
30
40
50
60
70
80
90
ψ - DEGREES

Figure V

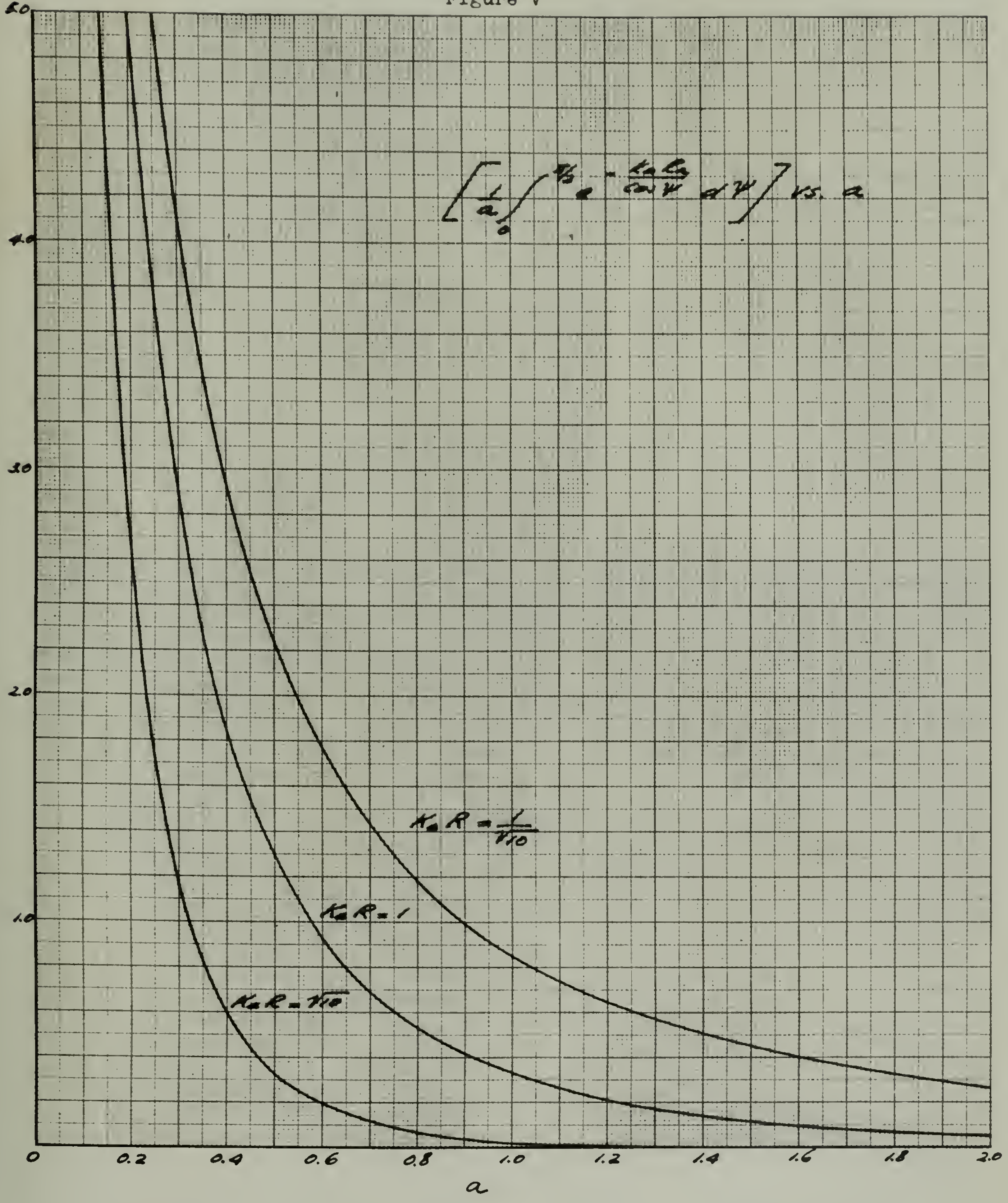

$\left[\frac{1}{a}\int_0^{\pi/2} e^{-\frac{K_a C_a}{\cos\psi}}\,d\psi\right]$ vs. a
$K_a R = \frac{1}{\sqrt{10}}$
$K_a R = 1$
$K_a R = \sqrt{10}$
5.0
4.0
3.0
2.0
1.0
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
a

Figure VI

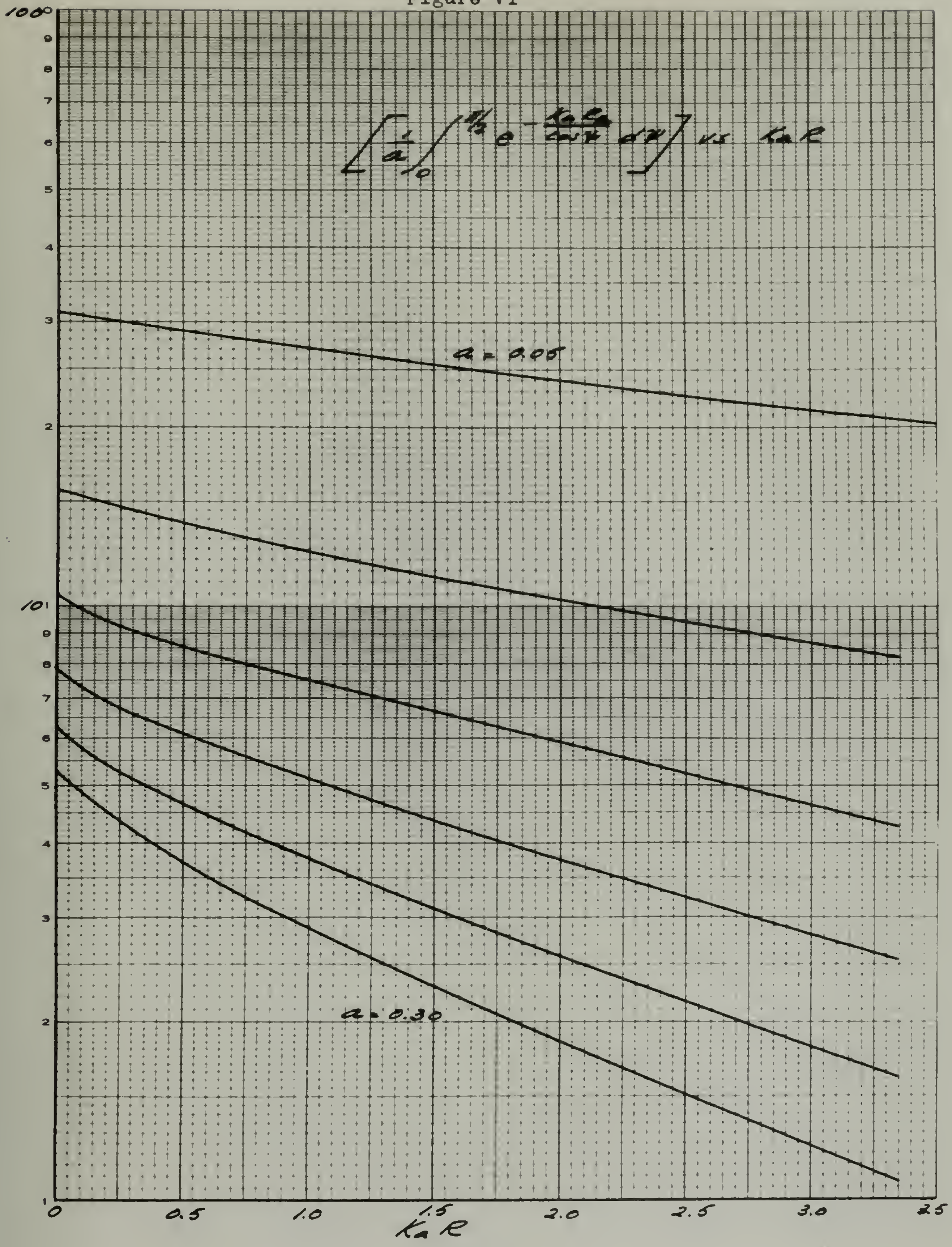

[1/a ∫₀^{π/2} e^{−K_a R / cos ψ} dψ] vs K_a R
a = 0.05
a = 0.30
100
9
8
7
6
5
4
3
2
10¹
9
8
7
6
5
4
3
2
1
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
K_a R

Figure VII

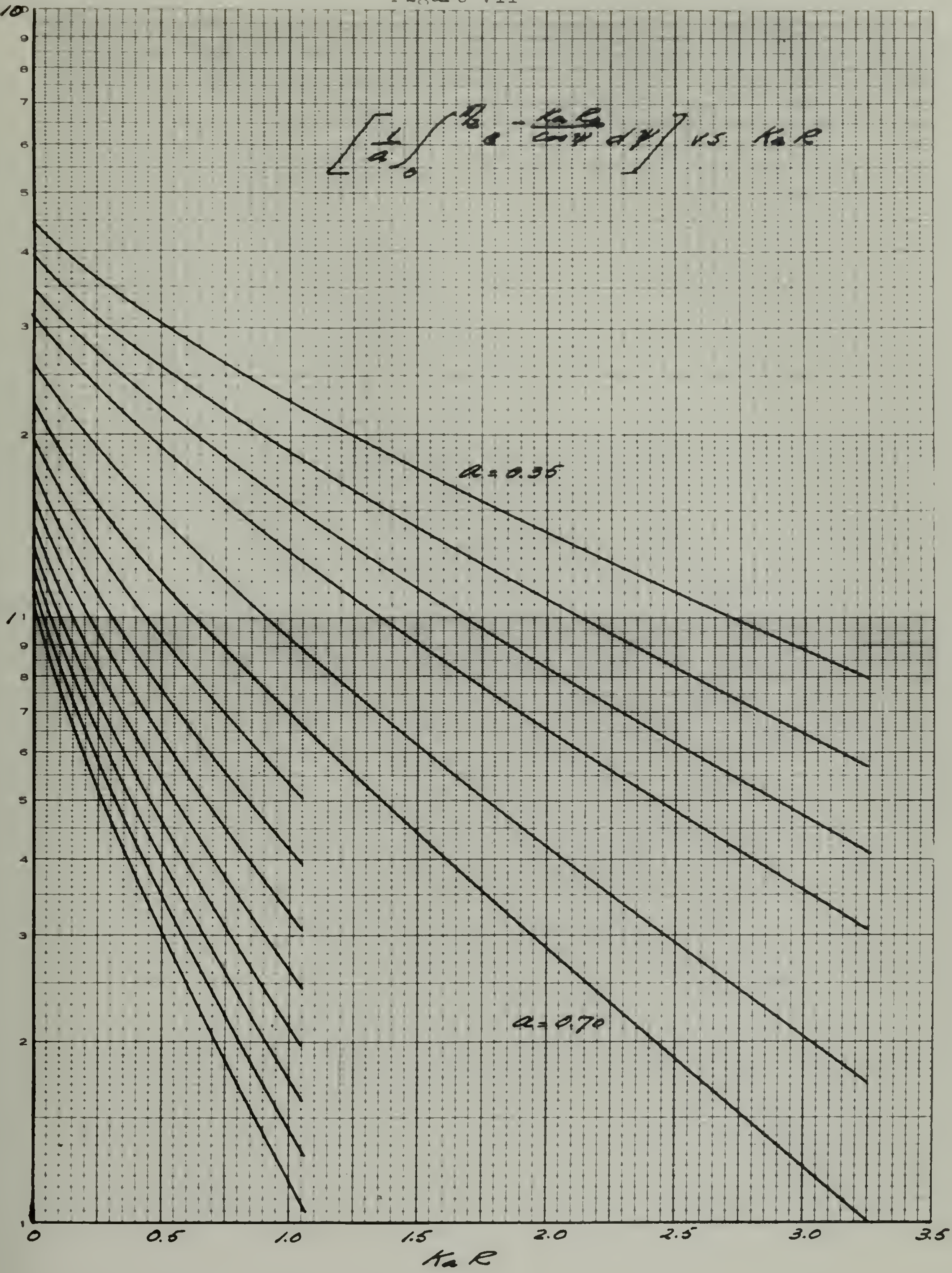
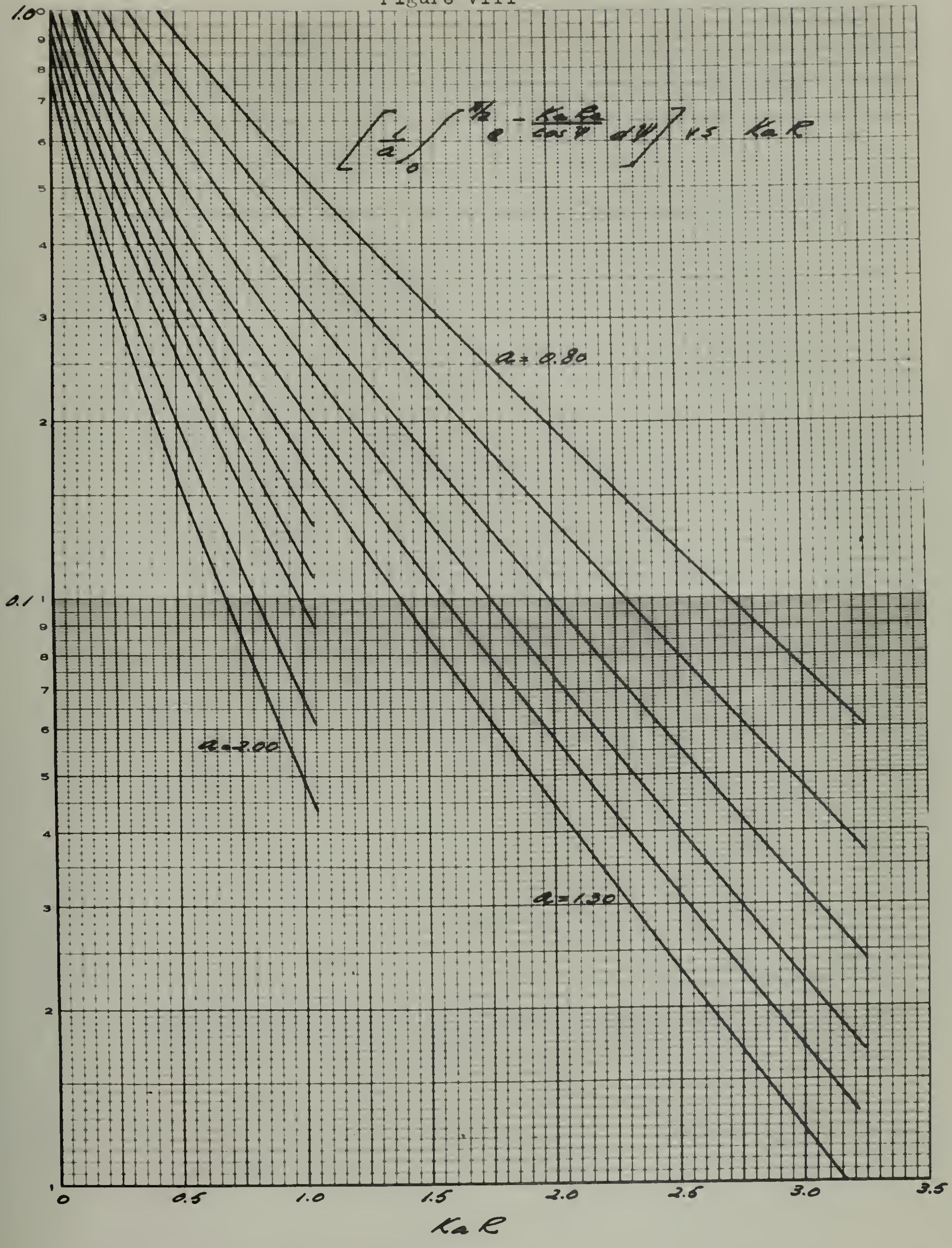

$\left[\frac{1}{a}\int_0^{\pi/2} e^{-\frac{K_a R}{\cos\psi}}\,d\psi\right]$ v.s. $K_a R$
$a = 0.35$
$a = 0.70$
10
9
8
7
6
5
4
3
2
1
9
8
7
6
5
4
3
2
1
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
$K_a R$

Figure VIII


$\left[\frac{1}{a}\int_0^{\pi/2} e^{-\frac{K_a R}{\cos\psi}}\,d\psi\right]$ vs $K_a R$
a = 0.80
a = 1.30
a = 2.00
1.0
9
8
7
6
5
4
3
2
0.1
9
8
7
6
5
4
3
2
1
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
Ka R

To accomplish this integration, carry out the following procedure:

(a) plot $\left[\frac{1}{a}\int_{\psi=0}^{\psi=\pi/2} e^{-\frac{K_a R a}{\cos\psi}} d\psi\right]$ versus (a) at constant values of $K_a R$ (Figure V).

(b) evaluate (a) as a function of θ_2 at given values of ($\frac{r_1}{R}$) and ($\frac{r_2}{R}$).

(c) plot $\left[\frac{1}{a}\int_{\psi=0}^{\psi=\pi/2} e^{-\frac{K_a R a}{\cos\psi}} d\psi\right]$ versus θ_2 as obtained from the previous plot at the corresponding values of (a), for given values of $K_a R$, $\frac{r_1}{R}$ and $\frac{r_2}{R}$. *

(d) carry out numerical integrations of the resulting plots from $\theta_2 = 0$ to $\theta_2 = \pi$.

The results of the integration for a sample value of $K_a R = 0.78$ are shown in Figure X plotted against ($\frac{r_2}{R}$) at constant values of ($\frac{r_1}{R}$).

Step 4

Determine the radiation $(dq)_j$ which is received by cube (1) from the whole gas in the pipe. That is, integrate the preceeding result across the radius ($\frac{r_2}{R}$) from 0 to 1.

$$(dq)_j = \left[\frac{4}{\pi}(K_e R)(K_a R)\left(\frac{\Delta v_1}{R^3}\right)\right]\int_{\left(\frac{r_2}{R}\right)=0}^{\left(\frac{r_2}{R}\right)=1}(E_2 R^2)\left(\frac{r_2}{R}\right)\left\{\int_{\theta_2=0}^{\theta_2=\pi}\left[\frac{1}{a}\int_{\psi=0}^{\psi=\pi/2} e^{-\frac{K_a R a}{\cos\psi}} d\psi\right] d\theta_2\right\} d\left(\frac{r_2}{R}\right) \quad (30)$$

---

*See Figure IX for a sample

Figure IX

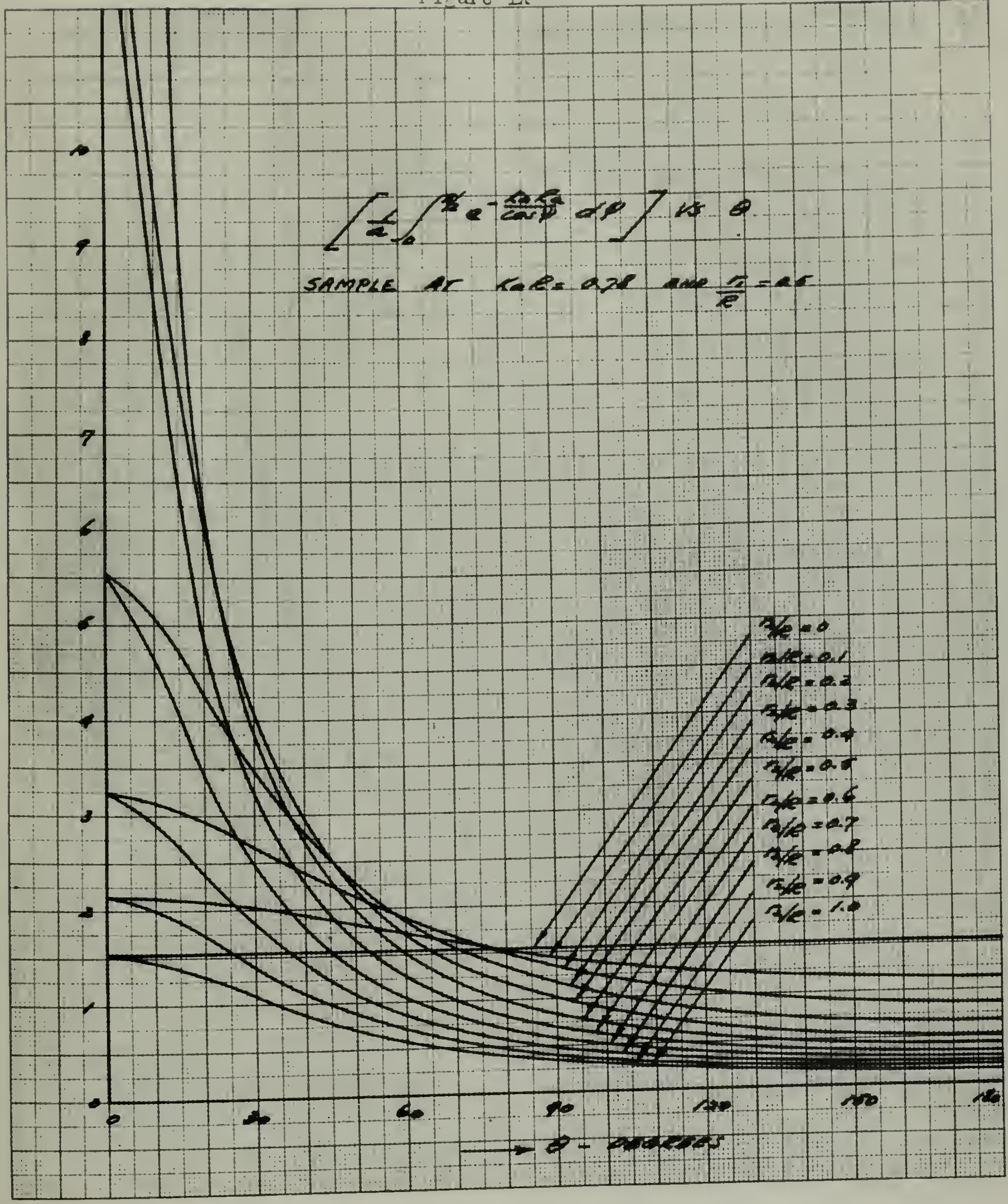
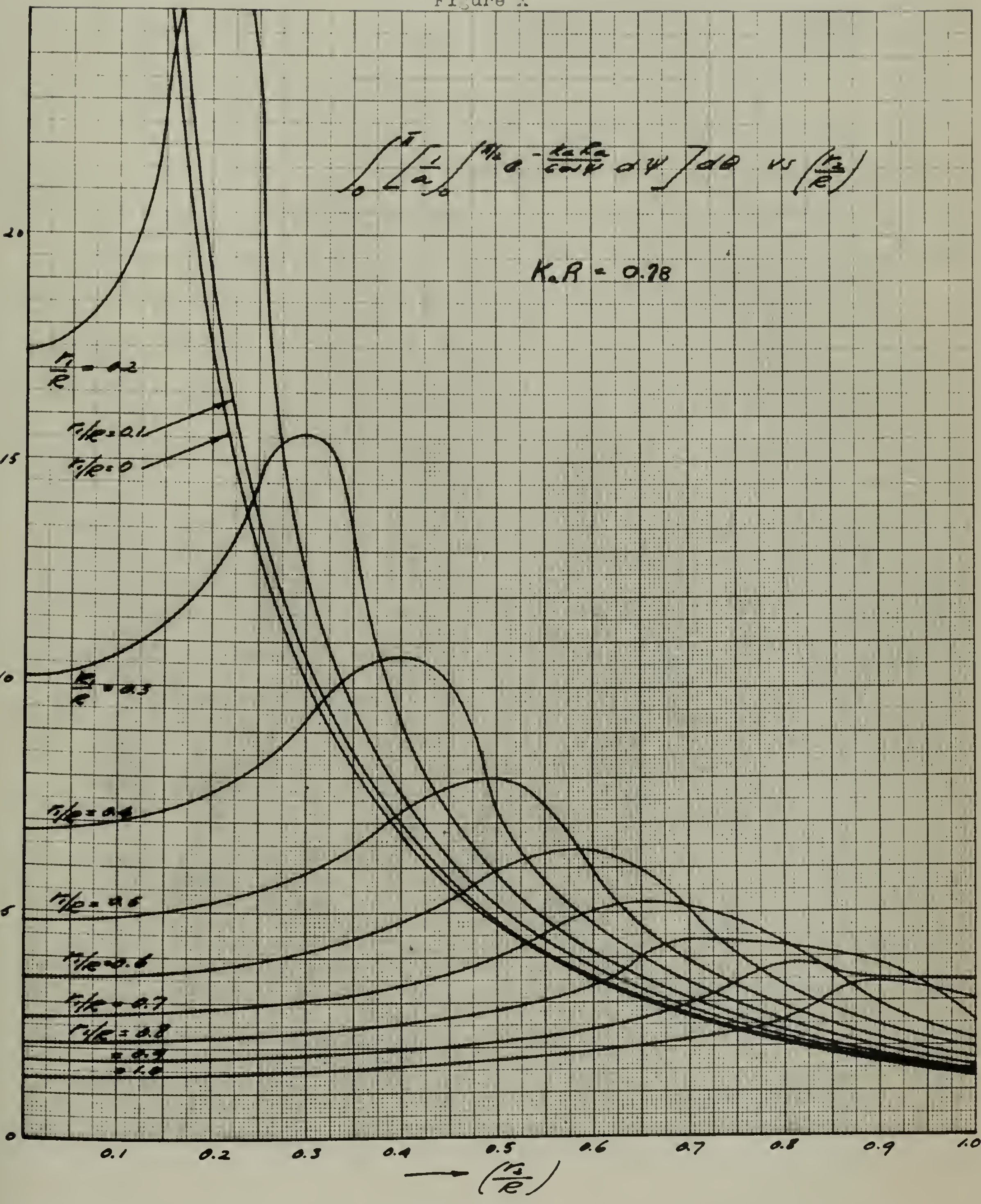

$\left[\frac{1}{\pi}\int_0^{\pi/2} e^{-\frac{r_2 K_a}{\cos\phi}}\,d\phi\right]$ vs θ
SAMPLE AT KaR = 0.78 AND $\frac{r_1}{R}$ = 0.5
r₂/R = 0
r₂/R = 0.1
r₂/R = 0.2
r₂/R = 0.3
r₂/R = 0.4
r₂/R = 0.5
r₂/R = 0.6
r₂/R = 0.7
r₂/R = 0.8
r₂/R = 0.9
r₂/R = 1.0
0
1
2
3
4
5
6
7
8
9
10
0
30
60
90
120
150
180
θ - DEGREES

Figure X


$\int_0^{\pi}\left[\frac{1}{a}\int_0^{\pi/2} e^{-\frac{k_a a}{\cos\psi}}\,d\psi\right]d\theta$ vs $\left(\frac{r_2}{R}\right)$
$K_a R = 0.78$
$\frac{r_1}{R} = 0.2$
$r_1/R = 0.1$
$r_1/R = 0$
$\frac{r_1}{R} = 0.3$
$r_1/R = 0.4$
$r_1/R = 0.5$
$r_1/R = 0.6$
$r_1/R = 0.7$
$r_1/R = 0.8$
$= 0.9$
$= 1.0$
0
5
10
15
20
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
$\longrightarrow \left(\frac{r_2}{R}\right)$

Note that $E_2 = \sigma(T_2)^4$ must be brought into this integral. Unfortunately, the T_2 distribution with ($\frac{r_2}{R}$) is usually an unknown for any given problem. This will lead to a trial and error reiteration procedure to establish the proper temperature distribution.

Step 5

Next consider the absorption of radiation from the whole pipe wall. First, evaluate the radiation emission $(d^3q)_w$ which is received by the given cube (1) from an area element on the pipe wall of dimensions $r_o d\theta_o$ by dx_o located at distance (z_o) at $r_o = R$, θ_o and x_o. (The pipe wall is assumed to be black.)

The one way radiation, $(d^3q)_w$, from the wall area element, (dA_o), to the gas cube (1), (dv_1), is equal to the emission from the wall element, $(4E_w dA_o)$, multiplied by the cosine of the angle between the connecting line and the normal to dA_o, $(\cos\beta)$, by the transmittance of the intervening gas $(e^{-K_a z_o})$ and by the absorptivity of the gas cube (1), $\frac{K_a dv_1}{4\pi z_o^2}$.

$$(d^3q)_w = \frac{(4E_w dA_o)(\cos\beta)(K_a dv_1)\, e^{-K_a z_o}}{4\pi z_o^2} \tag{31a}$$

For this case, (cosine β) can be shown to be *

$$(\cos\beta) = \frac{R - r_1\cos\theta_o}{z_o} \qquad \frac{R - r_1\cos\theta_o}{\sqrt{r_1^2 + R^2 - 2r_1R\cos\theta_o + x_o^2}}$$

---

* See Appendix II

Therefore

$$(d^3q)_w = \frac{4K_a E_w R\, d\theta_o\, dx_o (R - r_1\cos\theta_o)\, dv_1\, e^{-K_a\sqrt{r_1^2 + R^2 - 2r_1R\cos\theta_o + x_o^2}}}{4\pi\left[r_1^2 + R^2 - 2r_1R\cos\theta_o + x_o^2\right]^{3/2}} \qquad (31b)$$

which can be simplified in dimensionless form as in step 1 for the gas. So

$$(d^3q)_w = \left[\frac{1}{\pi}(K_aR)(E_wR^2)(d\theta_o)\left(\frac{dv_1}{R^3}\right)\right]\left[\frac{\left(1-\frac{r_1}{R}\cos\theta_o\right)e^{-K_aR\sqrt{a_o^2+X_o^2}}\, dX_o}{\left[a_o^2 + X_o^2\right]^{3/2}}\right] \qquad (32)$$

Step 6

Integrate this expression along the complete length of the pipe in order to obtain the radiation emission $(d^2q)_w$ which is received by cube (1) from a thin strip wall element extending the length of the pipe at $r_o = R$ and θ_o. Again, the axial variation in wall temperature is neglected for purposes of this integration.

$$(d^2q)_w = \left[\frac{2}{\pi}(K_aR)(E_wR^2)(d\theta_o)\left(\frac{dv_1}{R^3}\right)\left(1-\frac{r_1}{R}\cos\theta_o\right)\right]\int_{X_o=0}^{X_o=\infty}\frac{e^{-K_aR\sqrt{a_o^2+X_o^2}}\, dX_o}{\left[a_o^2 + X_o^2\right]^{3/2}} \qquad (33)$$

Again variables are changed in order to avoid infinity.

Let

$$a_o^2 = \frac{r_1}{R}^2 + 1 - 2\,\frac{r_1}{R}\cos\theta_o$$

$$X_o = \left(\frac{x_o}{R}\right)$$

and

$$d\psi_o = \frac{a_o\, dX_o}{a_o^2\ \ X_o^2}$$

Then

$$\left(d^2 q\right)_w = \left[\frac{2}{\pi}(K_a R)(E_w R^2)(d\theta_0)\left(\frac{dv_1}{R^3}\right)\left(1 - \frac{r_1}{R}\cos\theta_0\right)\right]\left[\frac{1}{a_0^2}\int_{\psi_0=0}^{\psi_0=\pi/2} e^{-\frac{K_a R a_0}{\cos\psi_0}} \cdot \cos\psi_0 \, d\psi_0\right] \quad (34)$$

This integral can be carried out numerically for a suitable range of values of ($K_a Ra_o$). The plot of $\left[e^{-\frac{K_a R a_0}{\cos\psi_0}} \cdot \cos\psi_0\right]$ versus ψ_0 at constant ($K_a Ra_o$) values is shown in Figure XI. The integrals of these curves multiplied by $\frac{1}{a_o^2}$ are plotted against (a_o) at constant value of ($K_a R$) in Figure XII. These curves are also cross-plotted on semi-log paper,

$$\text{Log}\left[\frac{1}{a_0^2}\int_0^{\pi/2} e^{-\frac{K_a R a}{\cos\psi_0}} \cdot \cos\psi_0 \, d\psi_0\right]$$ versus ($K_a R$) at constant (a_o).

Step 7

Determine the radiation emission $(dq)_w$ which is received by cube (1) from the whole pipe wall by integration of the preceeding expression around the circumference of the pipe.

$$\left(dq\right)_w = \left[\frac{4}{\pi}(K_a R)(E_w R^2)\left(\frac{dv_1}{R^3}\right)\right]\int_{\theta_0=0}^{\theta_0=\pi}\left(1 - \frac{r_1}{R}\cos\theta_0\right)\left[\frac{1}{a_0^2}\int_0^{\pi/2} e^{-\frac{K_a R a_0}{\cos\psi_0}} \cdot \cos\psi_0 \, d\psi_0\right] d\theta_0 \quad (35)$$

Note that $E_w = \sigma T_o^4$ is constant for this integration and remains outside the integral. However, (a_o) is a function of θ_o .

---

* See Figures XIII, XIV, and XV and XVI

Figure XI

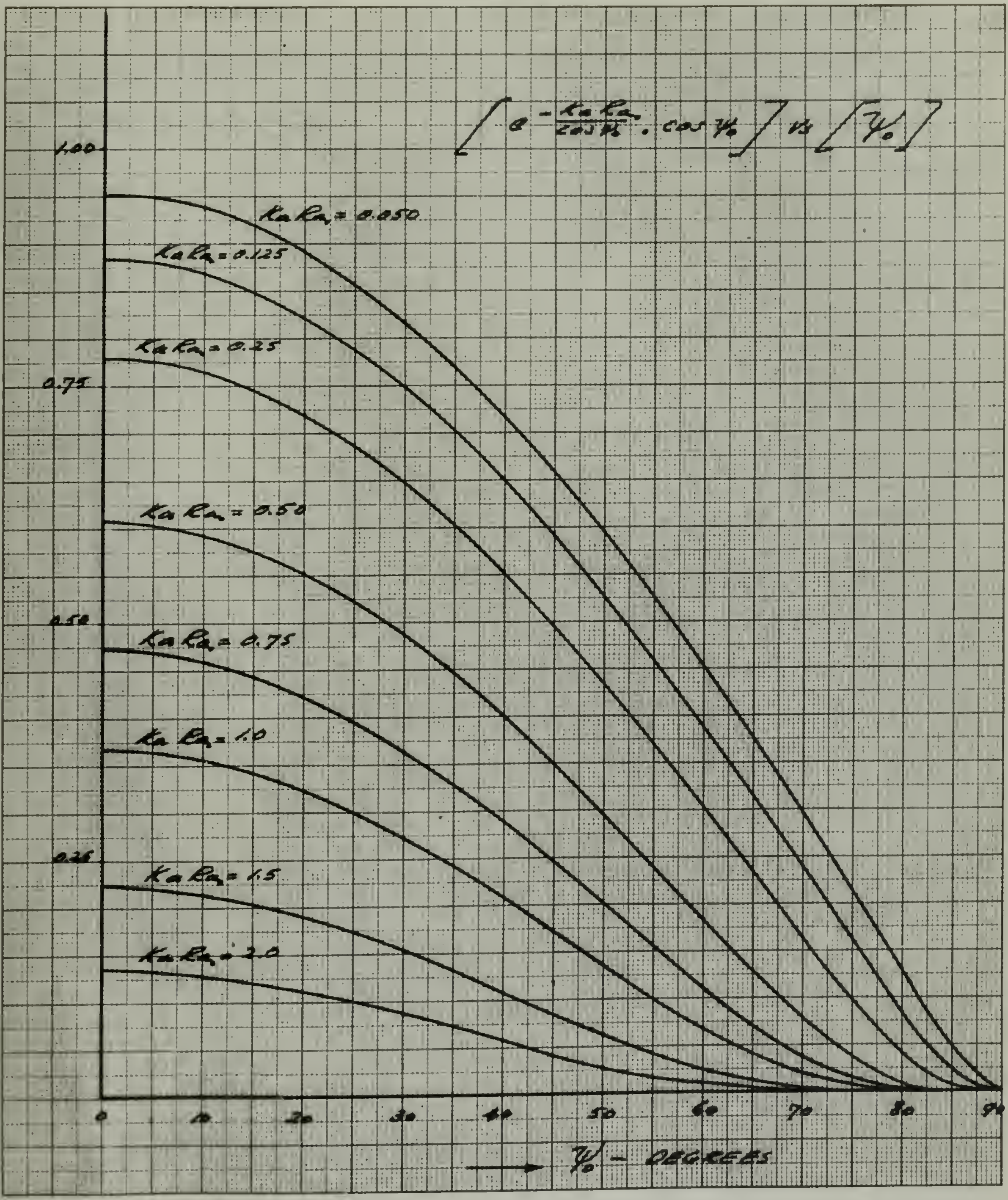

$\left[e^{-\frac{K_a K_m}{\cos\psi_0}} \cdot \cos\psi_0 \right]$ vs $\left[\psi_0\right]$
1.00
0.75
0.50
0.25
Ka Km = 0.050
Ka Km = 0.125
Ka Km = 0.25
Ka Km = 0.50
Ka Km = 0.75
Ka Km = 1.0
Ka Km = 1.5
Ka Km = 2.0
0
10
20
30
40
50
60
70
80
90
ψ_0 – DEGREES

Figure XII

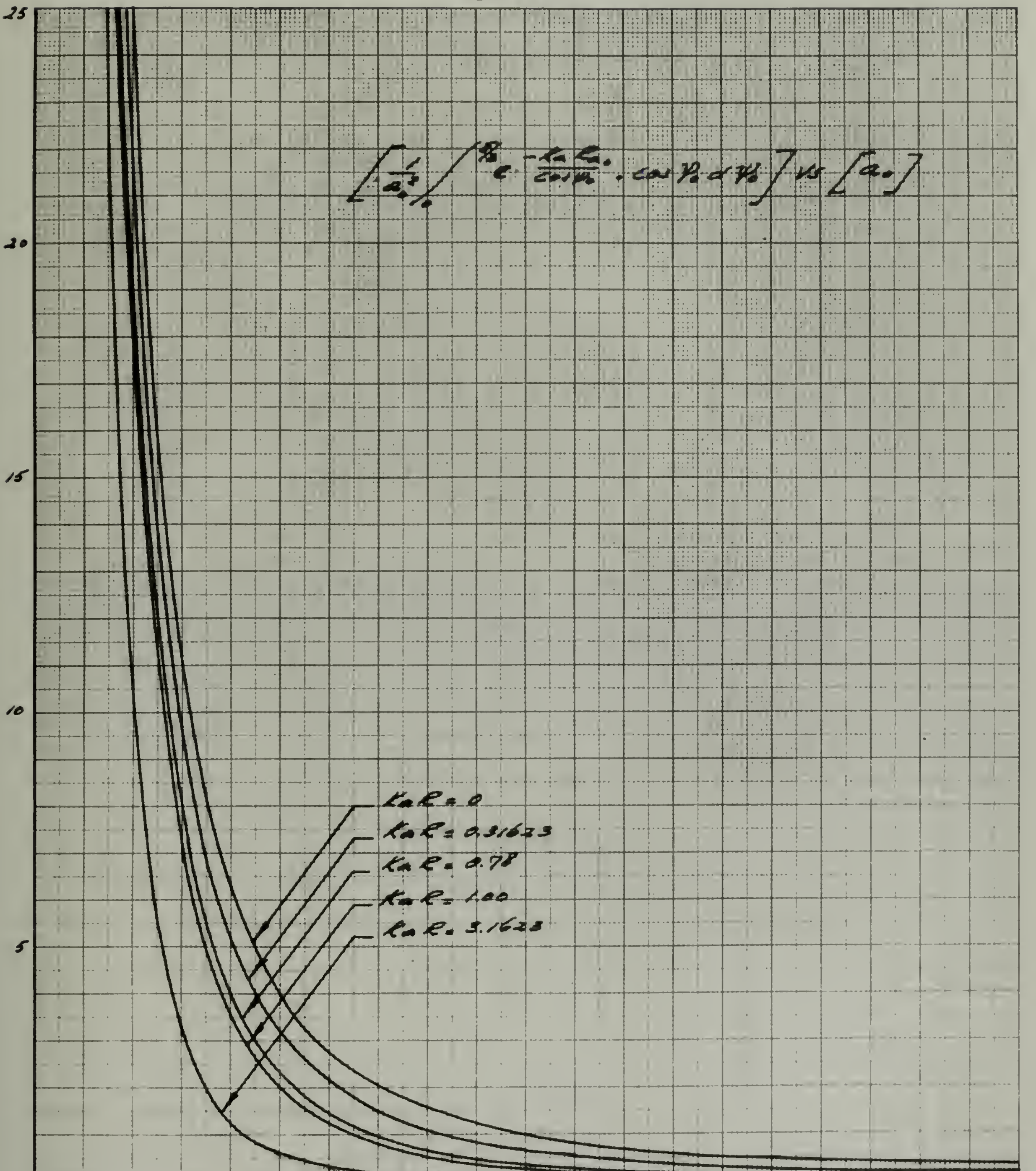

$\left[\frac{1}{a_0^2}\int_0^{\psi_0} e^{-\frac{k_a R_0}{\cos\psi_0}}\cdot\cos\psi_0\, d\psi_0\right]$ vs $[a_0]$
$k_a R = 0$
$k_a R = 0.31623$
$k_a R = 0.78$
$k_a R = 1.00$
$k_a R = 3.1623$
25
20
15
10
5
0
.50
1.00
1.50
2.00
$\longrightarrow a_0$

Figure XIII

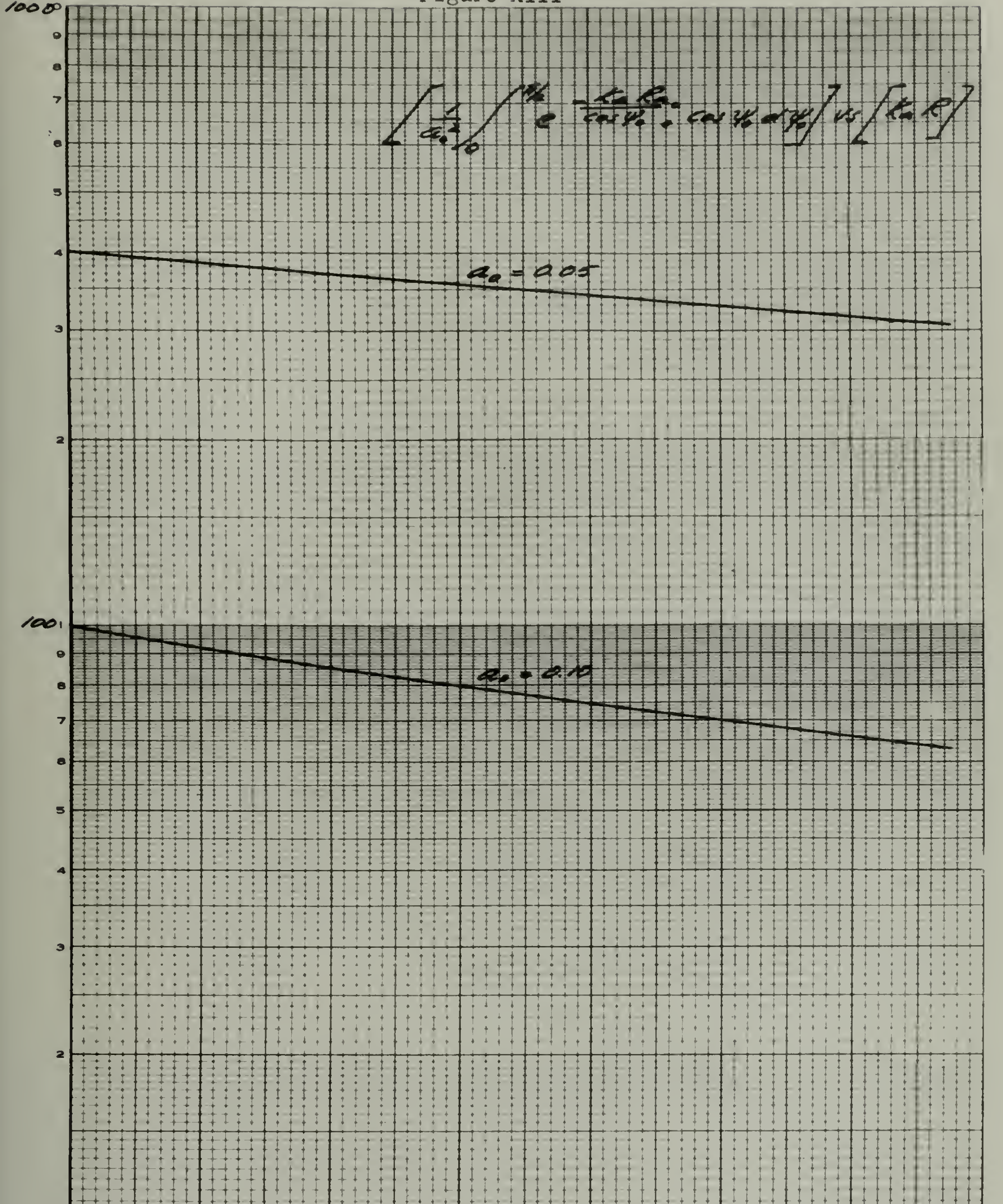

$\left[\frac{1}{a_0^2}\int_0^{\pi/2} e^{-\frac{k_a R_0}{\cos\psi_0}} \cdot \cos\psi_0 \, d\psi_0\right]$ vs. $\left[k_a R\right]$
$a_0 = 0.05$
$a_0 = 0.10$
1000
9
8
7
6
5
4
3
2
100
9
8
7
6
5
4
3
2
10
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
→ $k_a R$

Figure XIV

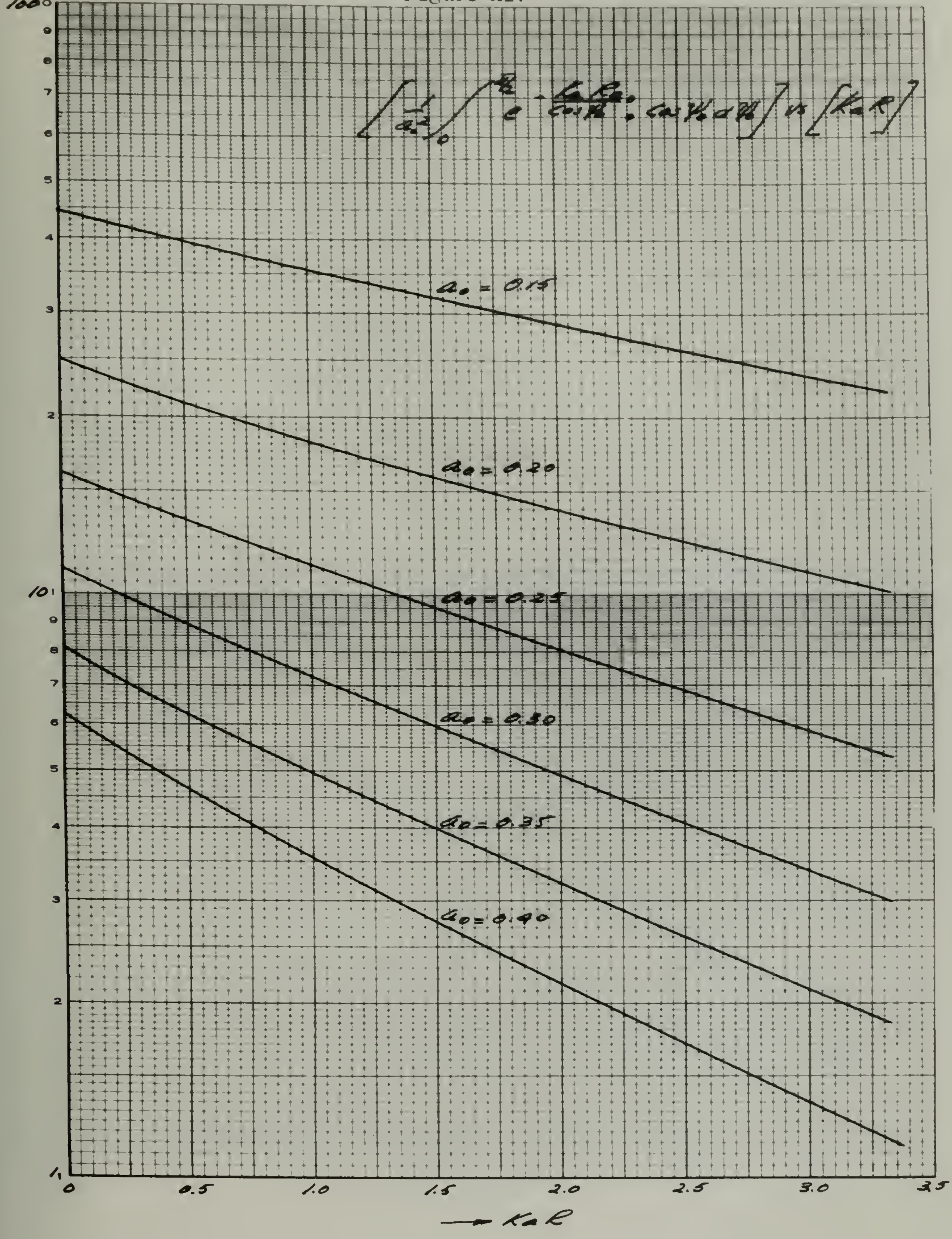

$\left[\frac{1}{a_0^2}\int_0^{\pi/2} e^{-\frac{K_a R a_0}{\cos\psi_0}} \cdot \cos\psi_0 \, d\psi_0\right]$ vs $\left[K_a R\right]$
$a_0 = 0.15$
$a_0 = 0.20$
$a_0 = 0.25$
$a_0 = 0.30$
$a_0 = 0.35$
$a_0 = 0.40$
10^0
9
8
7
6
5
4
3
2
10^1
9
8
7
6
5
4
3
2
1,
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
→ $K_a R$

Figure XV

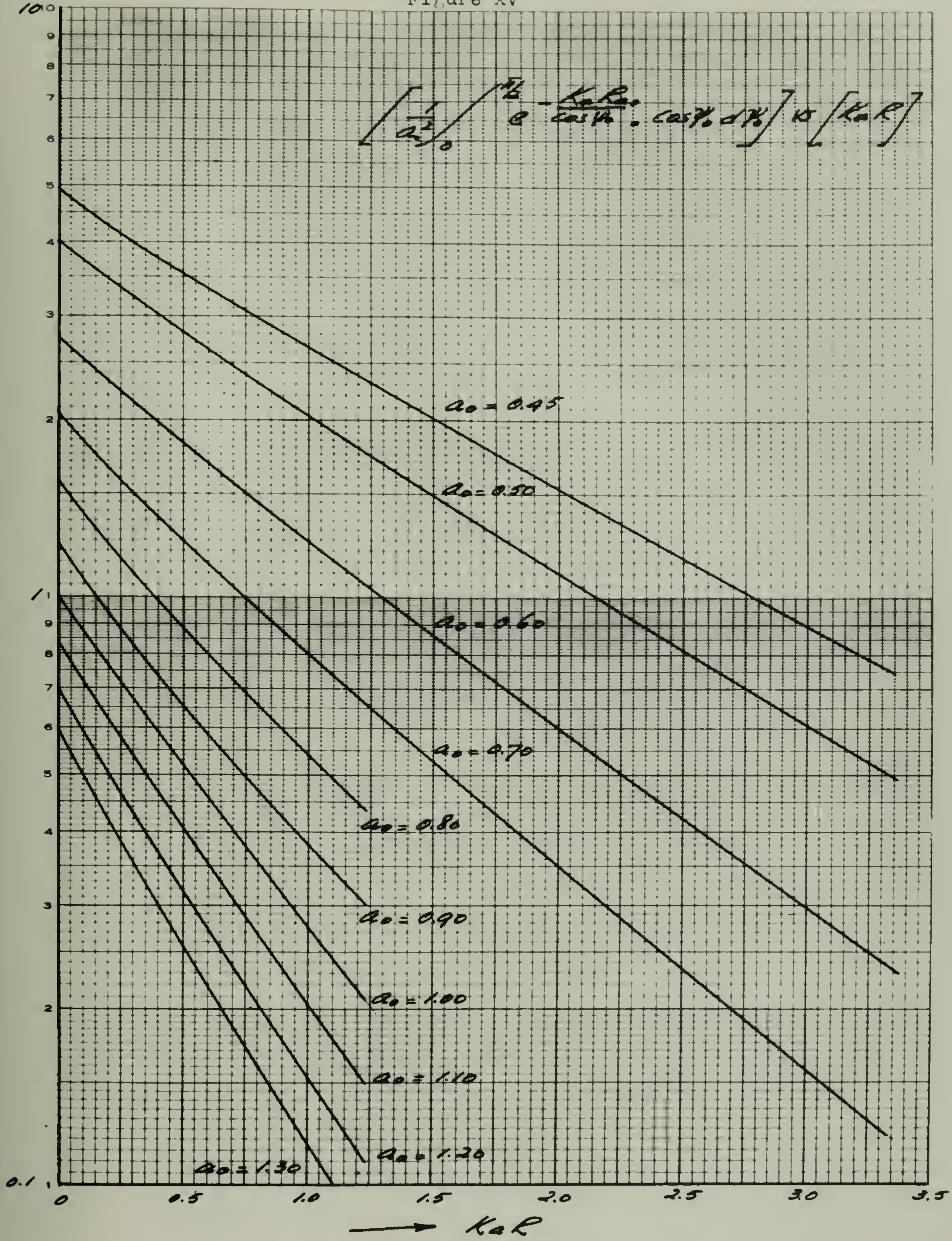
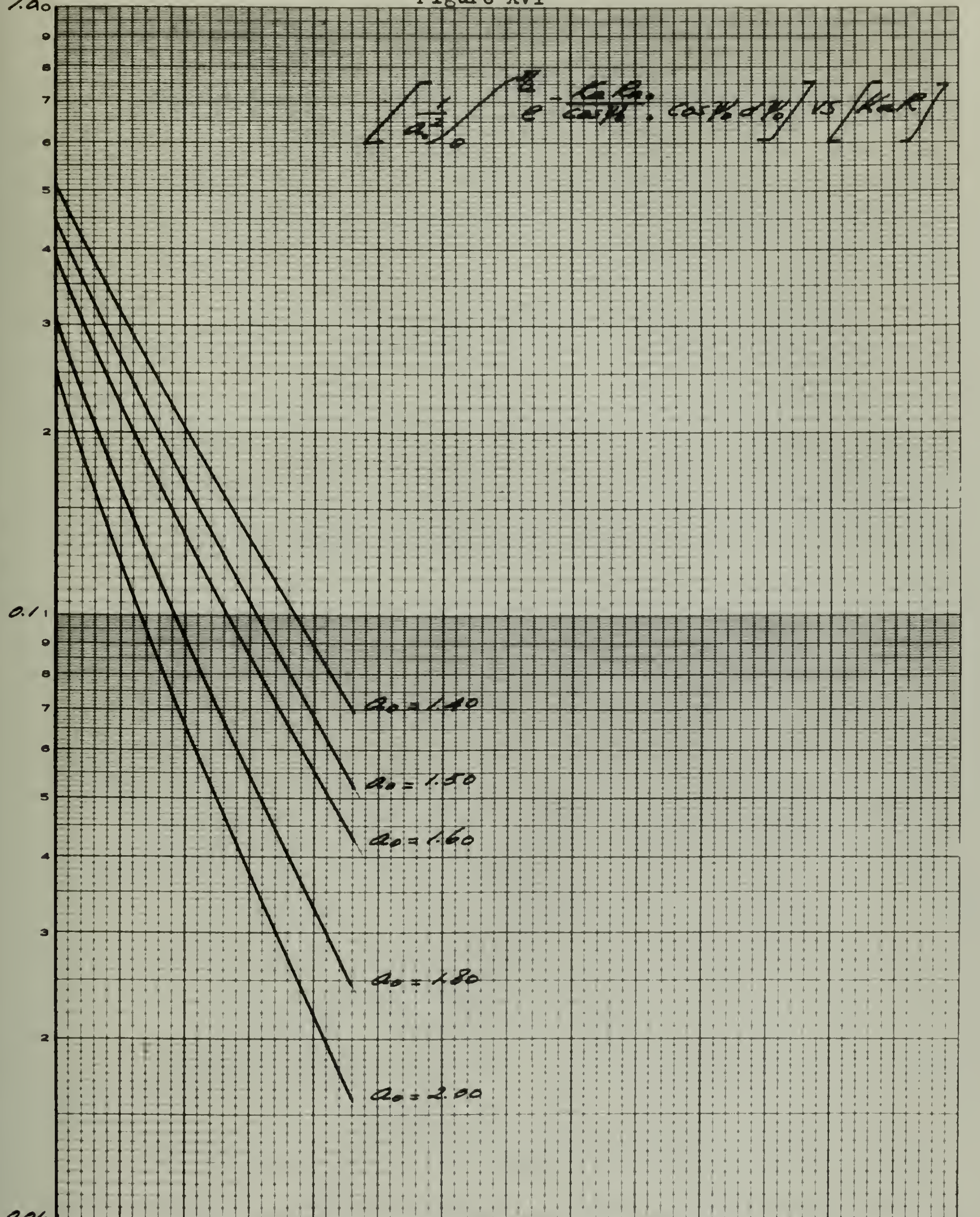

$\left[\frac{1}{a_o^2}\int_0^{\pi/2} e^{-\frac{K_a R_o}{\cos\psi_o}} \cdot \cos\psi_o \, d\psi_o\right]$ vs $\left[K_a R\right]$
$a_o = 0.45$
$a_o = 0.50$
$a_o = 0.60$
$a_o = 0.70$
$a_o = 0.80$
$a_o = 0.90$
$a_o = 1.00$
$a_o = 1.10$
$a_o = 1.20$
$a_o = 1.30$
10.0
9
8
7
6
5
4
3
2
1
9
8
7
6
5
4
3
2
0.1
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
$\longrightarrow K_a R$

Figure XVI


$\left[\frac{1}{a_0^2}\int_0^{\frac{\pi}{2}} e^{-\frac{k_a R_0}{\cos\psi_0}}\cdot\cos\psi_0\, d\psi_0\right]$ vs $[k_a R]$
1.00
9
8
7
6
5
4
3
2
0.1
9
8
7
6
5
4
3
2
0.01
$a_0 = 1.40$
$a_0 = 1.50$
$a_0 = 1.60$
$a_0 = 1.80$
$a_0 = 2.00$
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
→ $k_a R$

Therefore, the same procedure as used in the gas shell integration should be carried out here.

(a) plot $\left[\frac{1}{a_o^2}\int_0^{\pi/2} e^{-\frac{K_a R a_o}{\cos\psi_o}}\cdot\cos\psi_o\,d\psi_o\right]$ versus a_o at constant values of K_aR (Figure XII)

(b) evaluate a_o as a function of θ_o at given values of $\left(\frac{r_1}{R}\right)$

(c) Plot $\left(1 - \frac{r_1}{R}\cos\theta_o\right)\left[\frac{1}{a_o^2}\int_0^{\pi/2} e^{-\frac{K_a R a_o}{\cos\psi_o}}\cdot\cos\psi_o'\,d\psi_o\right]$ versus θ_o for given values of K_aR and $\left(\frac{r_1}{R}\right)$ *

(d) carry out numerical integration of the resulting plots from $\theta_o = 0$ to $\theta_o = \pi$.

The results of these integrations at a sample value of $K_aR = 0.78$ are shown in Figure XVIII plotted against $\left(\frac{r_1}{R}\right)$.

Step 8

Finally, determine the radiation emission $-(dq)_\epsilon$ from the cube (1). This one is simple.

$$(dq)_\epsilon = -4\,K_eE_1r_1dr_1d\theta_1dx_1 \tag{36}$$

or in dimensionless form

$$(dq)_\epsilon = -4(K_eR)(E_1R^2)\left(\frac{dv_1}{R^3}\right) \tag{37}$$

Step 9

Combine those radiation components from equations (30), (35), and (37) to determine the net radiation heat transfer to the cube (1)

$$(dq)_r = (dq)_g + (dq)_w + (dq)_\epsilon \tag{38}$$

---

*See Figure XVII

Figure XVII

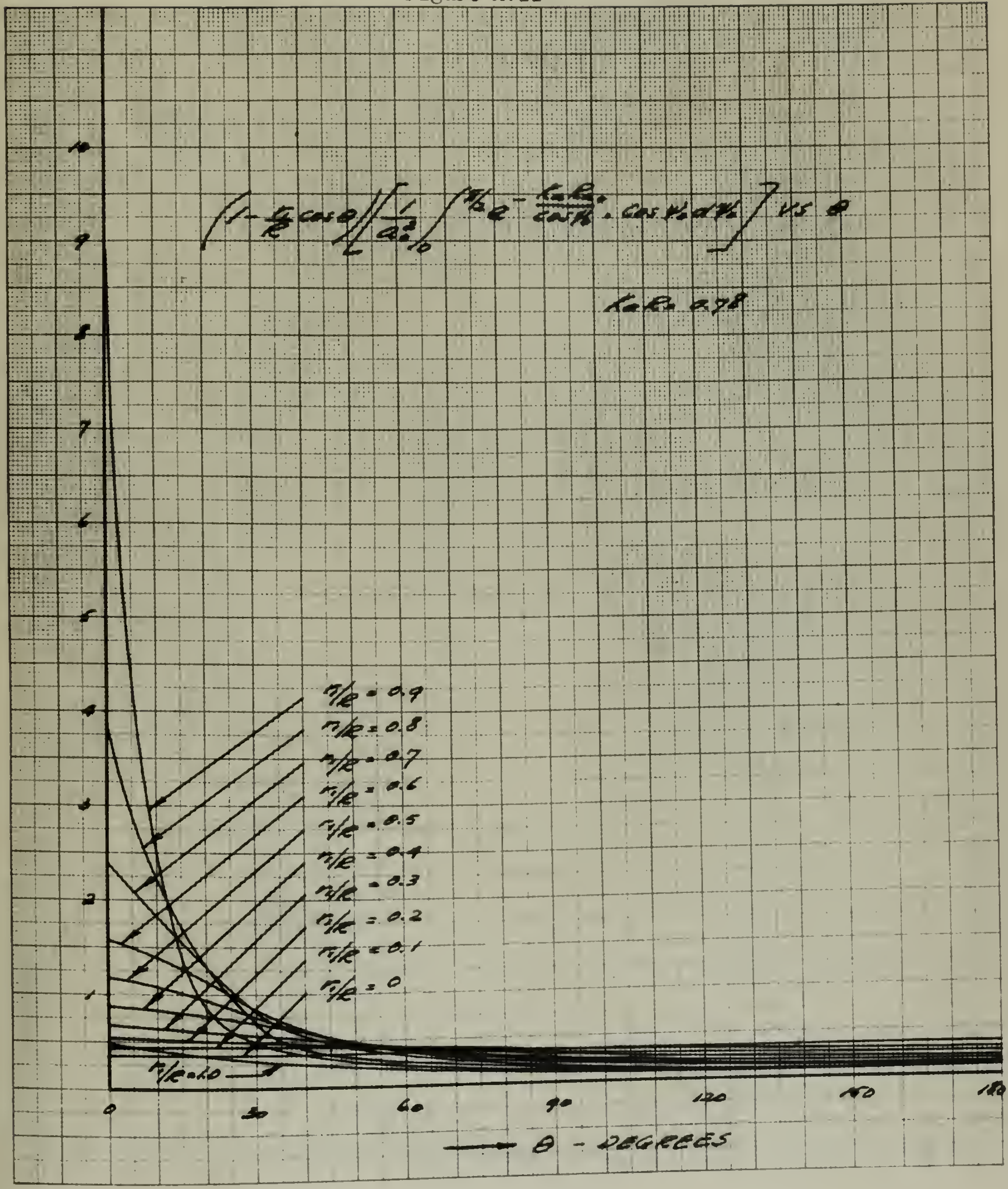
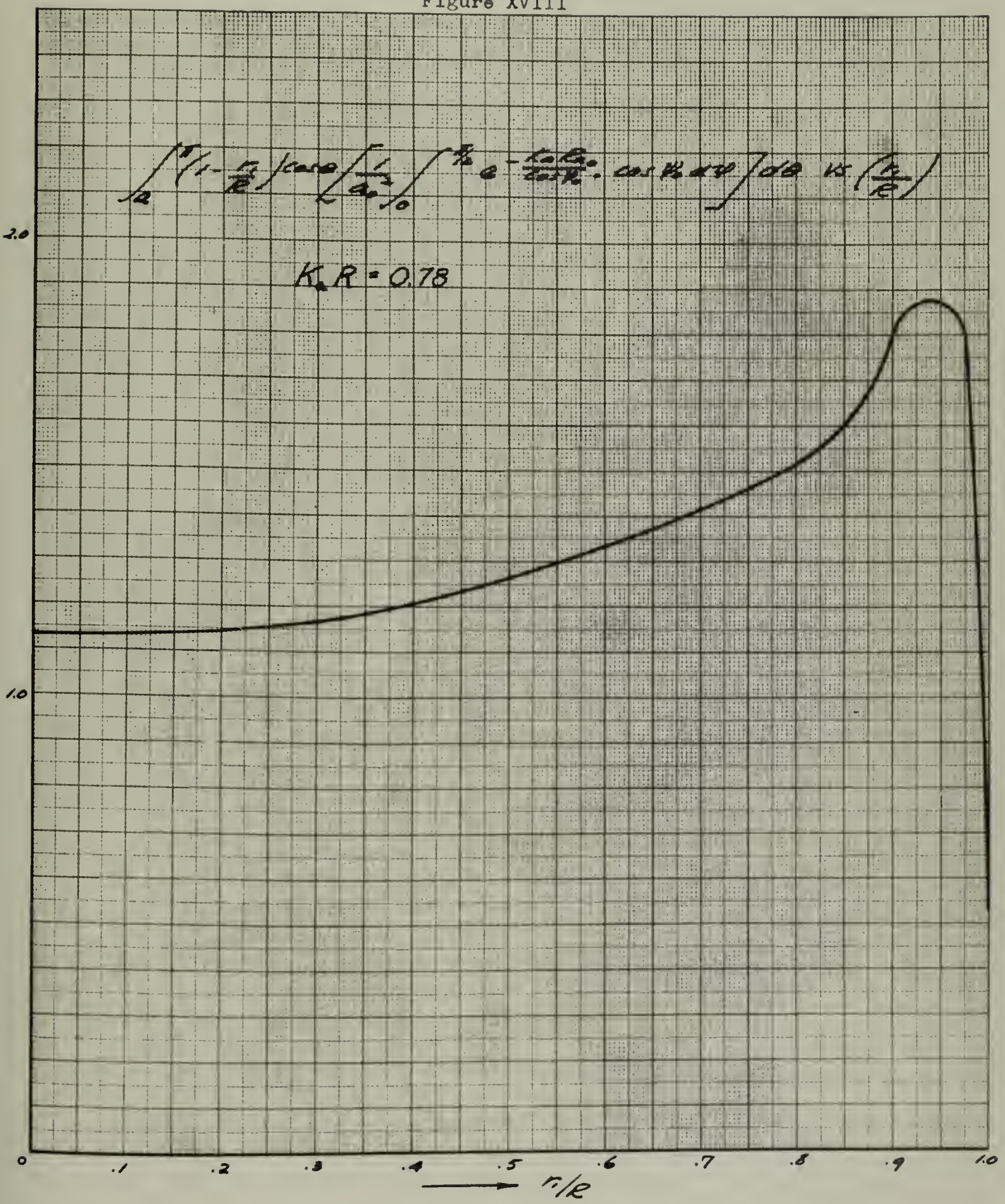

$\left(1-\frac{r}{R}\cos\theta\right) \Big/ \left[\frac{1}{a_0^2}\int_0^{\psi_0} e^{-\frac{K_a R_0}{\cos\psi_0}} \cdot \cos\psi_0\, d\psi_0\right]$ vs θ
$K_a R_0 = 0.78$
r/R = 0.9
r/R = 0.8
r/R = 0.7
r/R = 0.6
r/R = 0.5
r/R = 0.4
r/R = 0.3
r/R = 0.2
r/R = 0.1
r/R = 0
r/R = 1.0
10
9
8
7
6
5
4
3
2
1
0
30
60
90
120
150
180
θ - DEGREES

Figure XVIII



These terms should now be corrected for the fact that the gas is real and not a gray gas as assumed so far. Each term should be multiplied by the real gas transmittance weighting factor $(x)_g$. This factor may be thought of as that fraction of the total spectrum occupied by the absorption and emission of the real gas, so the remaining fraction $(1 - (x)_g)$ represents transparent non-absorbing and non-emitting gas. $(x)_g$ is evaluated as a function of the gas emissivities at a given P_gL value, (ϵ_g) and at a double P_gL value, (ϵ_{2g}).

$$(x)_g = \frac{\epsilon_g^2}{2\epsilon_g - \epsilon_{2g}}$$

Therefore, for the real gas

$$(dq)_r = (x)_g(dq)_g \quad (x)_g(dq)_w \quad (x)_g(dq)_\epsilon \tag{39}$$

Now the total net radiation heat transfer crossing the boundaries of this cube per unit of its area normal to the pipe wall is $\left(\frac{dq}{r_1 d\theta_1 dx_1}\right)_r = \left(\frac{dq}{dA}\right)_r$. Since this cube is typical of all cubes in a ring control volume at constant radius r_1, this expression also represents the net radiation heat flux to the ring control volume.

This now represents the net radiation heat flux to a ring control volume of real gas of thickness $d\left(\frac{r_1}{R}\right)$. To obtain the net radiation heat flux per unit dimensionless control volume thickness, simply divide by $d\left(\frac{r_1}{R}\right)$. This gives the required radiation point function, $(q/A)_r$, which is

the net radiation heat transfer to a control volume per unit area normal to the wall for a dimensionless unit thickness of control volume.[*] Then

$$(q/A)_r = (q/A)_{gas} + (q/A)_{wall} + (q/A)_{emission} \tag{40}$$

where

$$(q/A)_{gas} = \left[\frac{4(x)}{\pi} g(K_e R)(K_a R)\right] \int_{r_2/R=0}^{r_2/R=1} E_2\left(\frac{r_2}{R}\right) \left\{ \int_{\theta_2=0}^{\theta_2=\pi} \left[\frac{1}{a} \int_0^{\pi/2} e^{-\frac{K_a R a}{\cos\psi}} d\psi \right] d\theta_2 \right\} d\left(\frac{r_2}{R}\right) \tag{41}$$

$$(q/A)_{wall} = \left[\frac{4(x)}{\pi} g(K_a R)(E_w)\right] \int_{\theta_0=0}^{\theta_0=\pi} \left(1 - \frac{r_1}{R}\cos\theta_0\right) \left[\frac{1}{a_0^2} \int_0^{\pi/2} e^{-\frac{K_a R a_0}{\cos\psi_0}} \cdot \cos\psi_0 \, d\psi_0 \right] d\theta_0 \tag{42}$$

$$(q/A)_{emission} = -4(x)_g (K_e R) E_1 \tag{43}$$

PHASE C - Numerical Temperature Calculation for a Specific Example

In order to illustrate the application of the preceding results to an actual problem, a specific example will be solved numerically.

The statement of the problem is as follows:

First, consider a non-radiating hypothetical hot gas turbulently flowing in a long cool pipe. This non-radiating

---

[*] The significance of the selection of this term as the radiation function, $(q/A)_r$, which forms a part of $(q/A)_{app}$ in equation (7) is discussed in detail in the Discussion of Results.

gas has otherwise identical fluid properties as a radiating gas, pure carbon dioxide. The total heat transfer by convection from this hypothetical gas to the tube wall will be calculated under the following physical conditions.

(a) Pipe diameter, D = 2 inches

(b) Reynolds Number of flow,

$$N_{Re} = \frac{\rho V D}{g_o \mu} = 20{,}000$$

(c) Gas pressure p = 1 atmosphere

(d) Pipe wall temperature,

$$T_o = 550^{\circ}R$$

(e) Bulk gas temperature,

$$T_b = 2000^{\circ}R$$

where T_b may be defined as

$$\frac{T_o - T_{\text{¢}}}{T_o - T_b} = \frac{\int_0^{r_o} \frac{u_x}{(u_x)_{MAX.}} \left(\frac{T_o - T}{T_o - T_{\text{¢}}} \right) r\,dr}{\int_0^{r_o} \frac{u_x}{(u_x)_{MAX.}} \, r\,dr} \qquad [5] \quad (44)$$

(f) Gas film temp

$$T_f = \frac{T_b - T_o}{2} = 1275^{\circ}R$$

All physical properties are evaluated at the film temperature. These are tabulated in Appendix III.

From the von Karman solution, the coefficient of heat transfer (h) by convection only is determined from the

Stanton number.

$$N_{St} = \frac{h}{c_p G_m} = \frac{\alpha f/2}{1 + \sqrt{f/2}\left[5(\alpha N_{Pr} - 1) + 5 \ln(1 + 5\alpha N_{Pr}) - 5 \ln 6\right]} \qquad (45)$$

Then the net heat flux at the wall is determined

$$(q/A)_o = h\,(T_o - T_b) \qquad (46)$$

For this case

$$(q/A)_o = -\ 14{,}500 \text{ Btu/hr.ft}^2$$

Next, the temperature distribution across the pipe is determined by use of the von Karman method of solution. This solution is identical with the combined heat transfer temperature distribution solution presented by equations (20), (22) and (24) when $(q/A)_r = 0$.

The resulting temperature distribution from these equations for zero radiation is shown in Figure XIX.

Now shift to the case of the real radiating gas, pure carbon dioxide, with otherwise identical fluid properties. Assume that the following physical conditions exist.

(a) Pipe diameter - same value, $D = 2$ inches

(b) Reynolds number of flow - same value,

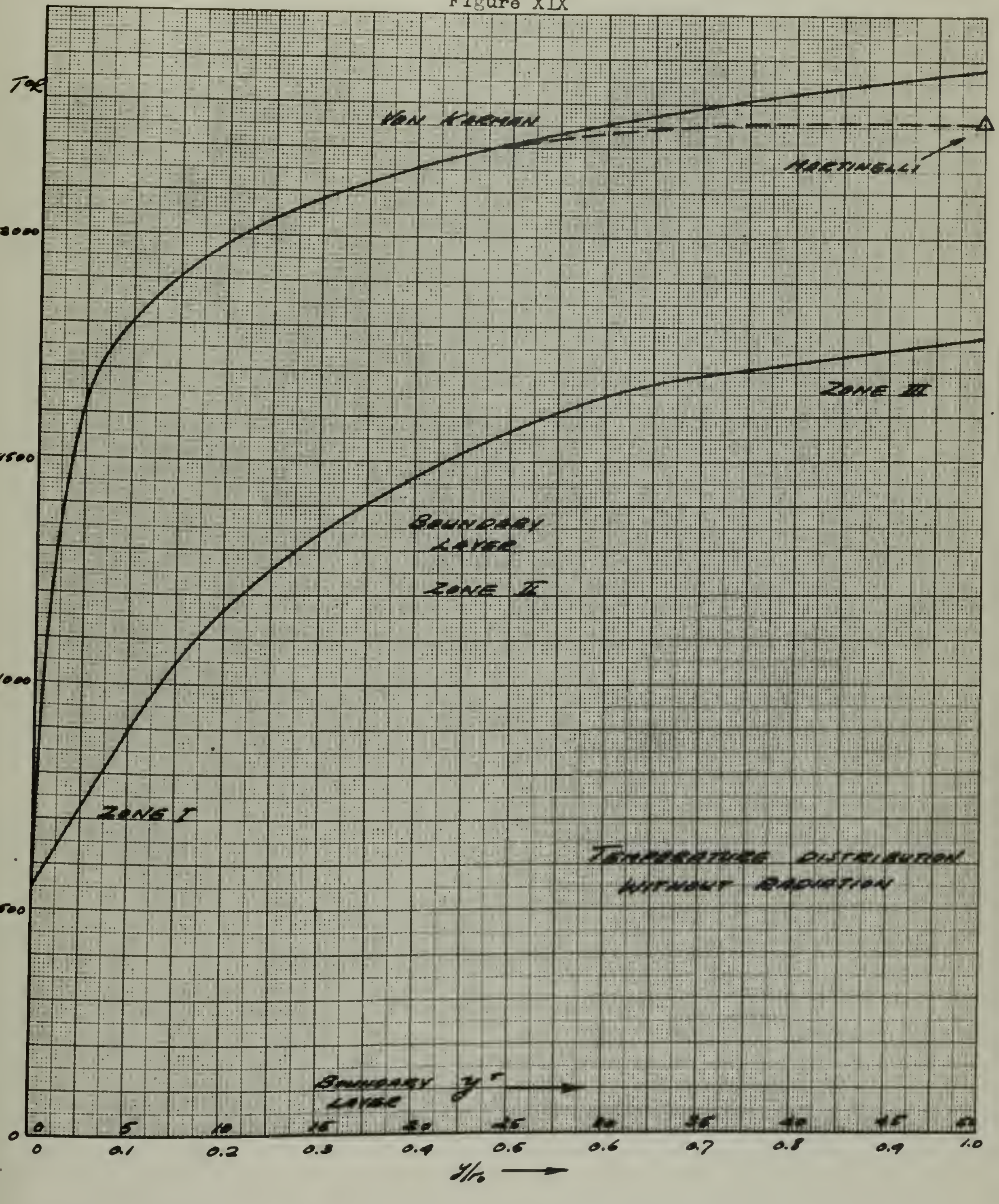
$N_{Re} = 20{,}000$

(c) Gas pressure - same value, $p = 1$ atmosphere

(d) Pipe wall temperature - same value, $T_o = 350^{\circ}$

(e) Total net heat flux for the gas at the wall (now including radiation) - same value, $(q/A)_o = -\ 14{,}500$ Btu/hr.ft^2 (not the same as the net heat flux to the wall as shown in Phase I.)

Figure XIX


T°R
2000
1500
1000
500
0
Van Karman
Martinelli
Zone III
Boundary Layer
Zone II
Zone I
Temperature Distribution Without Radiation
Boundary Layer
y^+ →
0
5
10
15
20
25
30
35
40
45
50
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
y/r_0 →

All physical properties are assumed to be the same value. Radiation properties will be evaluated at the bulk gas temperature.

In this case, the temperature distribution and the bulk fluid temperature are unknown. It is assumed that the bulk temperature will not change radically, so radiation properties will be evaluated at the same bulk temperature value $T_b = 2000°R$. These values are tabulated in Appendix [illegible].

The objective, at this point, is to determine a new temperature distribution including the radiation effects. This result can then be compared with the zero-radiation solution for comparative analysis of the effect of radiation on the overall problem.

The solution for this new temperature distribution will require a reiteration procedure in order to apply the radiation terms in the problem. As a first estimate, the zero radiation temperature distribution will be used.

It should be noted that, for this problem, the first half of each of the three temperature distribution equations (20), (22) and (24) will not be affected by changes in the temperature distribution. These $(q/A)_o$ terms may be evaluated once and for all at the given properties and $(q/A)_o$ value. In fact, these terms have already been evaluated in the zero-radiation solution. These results are tabulated in Appendix II.

The second part of each of these equations, involving the radiation flux $(q/A)_r$, does depend on the temperature distribution. The terms outside the integrals may be evaluated at the given properties. These coefficients are also tabulated in Appendix IV.

This brings up the big problem of numerical evaluation of radiation $(q/A)_r$ integral terms. The first job is the determination of $(q/A)_r$ as a function of radius using the selected first estimate temperature distribution.

This is the start of the reiteration procedure to establish a final compatible temperature distribution. Each of the following steps will be repeated until that temperature distribution is reached. Sample data and calculations for each of these steps, based on the final trial temperature distribution, are shown in Appendix V.

Step 1

Calculate the $E = \sigma T^4$ distribution with radius.

Step 2

Calculate the control volume total radiation emission flux as a function of radius.

$$(q/A)_{emission} = -4(x)_s(K_eR)E_1 \qquad (43)$$

The calculations of K_eR, K_aR and $(x)_s$ are shown in Appendix III.

Step 3

Calculate the wall contribution radiation flux as a function of radius from equation (42). These results will be constant for this particular problem since T_w is held constant as a given boundary condition.

Step 4

Calculate the gas contribution radiation flux as a function of radius from equation (41). This will require an integration for each radius point since E_2 is in the integral.

Step 5

Combine terms to determine the net radiation flux at the control volume:

$$(q/A)_r = (q/A)_{gas} + (q/A)_{wall} + (q/A)_{emission} \tag{40}$$

Step 6

Plot the results of step 5 on a large scale plot of $(q/A)_r$ versus radius and fare in a curve to determine more complete $(q/A)_r$ radial distribution data - especially near the wall.

Step 7

Calculate the $(q/A)_r$ terms inside the integrals of the three temperature distribution equations (20), (22) and (24).

Step 8

Plot the results of step 7 versus (y^+) and integrate those curves in steps adequate to define the temperature distribution.

Step 9

Multiply the resulting integrals by their appropriate coefficients previously calculated and tabulated in Appendix IV. This results in the complete evaluation of the $(q/A)_r$ radiation heat flux terms in the temperature distribution equations (20), (22) and (24).

Step 10

Combine these radiation heat flux terms with the $(q/A)_o$ terms tabulated in Appendix IV and so obtain the temperature distribution with radius from the appropriate equation:

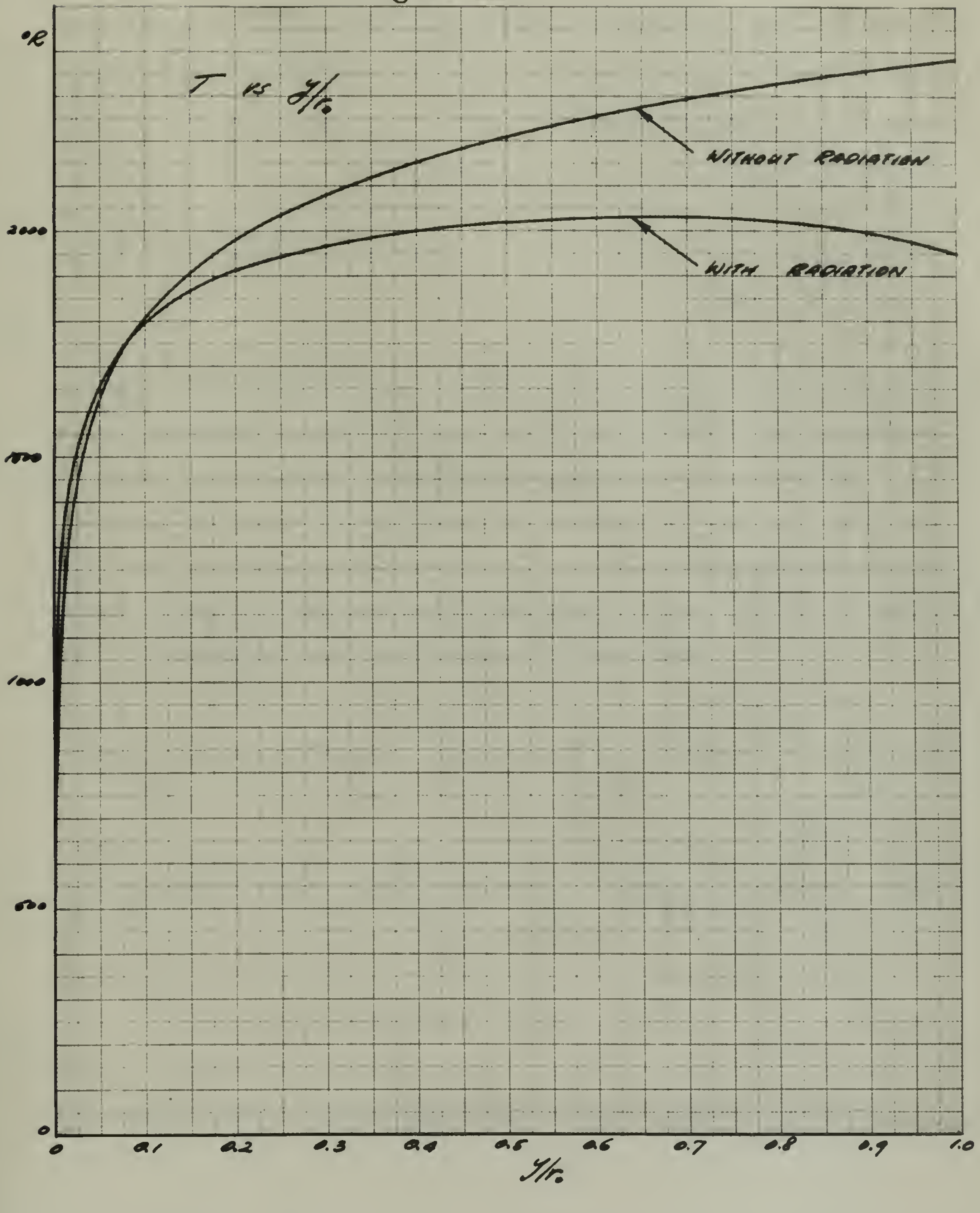
For $(0 < y^+ < 5)$ use equation (20).
For $(5 < y^+ < 30)$ use equation (22).
For $(30 < y^+ < r_o^+)$ use equation (24).

These ten steps are repeated until the resulting temperature distribution is nearly identical with the entering temperature distribution of that trial. This particular sample problem required eight trial solutions in order to reach convergence within plus or minus one degree for all radii except the centerline position. The final temperature distribution is plotted with the original zero trial temperature distribution for comparison in Figure XI.

PHASE D - Net Radiation Heat Transfer to the Wall

The next step in the heat transfer analysis is the calculation of the overall net radiation heat flux which actually reaches the wall from the whole gas. Now that the

Figure XX


°R
T vs y/r₀
WITHOUT RADIATION
WITH RADIATION
2000
1500
1000
500
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
y/r₀

temperature distribution in the gas is established, this is a relatively simple process.

The one way gas radiation analysis is the same as previously evaluated for the wall radiation received by a gas cube in steps 5 to 7 of Phase B, except for some changes in subscripts and the addition of one more integration across the radius of the pipe.

Step 1

Evaluate the radiation emission $(d^4q)_{w_g}$ which is received by a pipe wall area element dA_o from a small cube of gas of dimensions $r_1 d\theta_1$ by dr_1 by dx_1 located at r_1, θ_1 and x_1 .

$$(d^4q)_{w_g} = \frac{(4E_1 K_e r_1 d\theta_1 dr_1 dx_1) dA_o (R - r_1 \cos\theta_1)\, e^{-K_a\sqrt{r_1^2 + R^2 - 2r_1 R\cos\theta_1 + x_1^2}}}{4\pi(r_1^2 + R^2 - 2r_1 R \cos\theta_1 + x_1^2)^{3/2}} \quad (47)$$

which can be simplified in dimensionless form to the following:

$$(d^4q)_{w_g} = \left[\frac{L}{\pi}(K_e R)(E_1 R^2)\left(\frac{r_1}{R}\right) d\left(\frac{r_1}{R}\right) d\theta_1 \left(1 - \frac{r_1}{R}\cos\theta_1\right)\left(\frac{dA_o}{R^2}\right)\right] \frac{e^{-K_a R\sqrt{a_o^2 + X_1^2}}\, dX_1}{\left[a_o^2 + X_1^2\right]^{3/2}} \quad (48)$$

Step 2

Integrate this expression along the complete length of the pipe to obtain the radiation emission $(d^3q)_{w_g}$ which reaches the wall element from a long thin rod element of gas located at r_1 and θ_1 . The axial temperature variation is

neglected for this integration. Variables are changed to avoid infinity.

$$(d^3q)_{w_g} = \left[\frac{2}{\pi}(K_eR)(E_1R^2)\left(\frac{r_1}{R}\right)d\left(\frac{r_1}{R}\right)\left(1-\frac{r_1}{R}\cos\theta_1\right)d\theta_1\left(\frac{dA_0}{R^2}\right)\right]\frac{1}{a_0^2}\int_{\psi_0=0}^{\psi_0=\pi/2} e^{-\frac{K_aRa_0}{\cos\psi_0}}\cdot\cos\psi_0\,d\psi_0 \quad (49)$$

This integral has been carried out as previously discussed. The results are shown in Figures XI through XVI.

Step 3

Determine the radiation emission $(d^2q)_{w_g}$ which is received by the wall element from a thin cylindrical gas shell located at r_1 by integrating the preceding expression in θ_1

$$(d^2q)_{w_g} = \left[\frac{4}{\pi}(K_eR)(E_1R^2)\left(\frac{r_1}{R}\right)d\left(\frac{r_1}{R}\right)\left(\frac{dA_0}{R^2}\right)\right]\int_{\theta_1=0}^{\theta_1=\pi}\left(1-\frac{r_1}{R}\cos\theta_1\right)\left[\frac{1}{a_0^2}\int_0^{\pi/2} e^{-\frac{K_aRa_0}{\cos\psi_0}}\cdot\cos\psi_0\,d\psi_0\right]d\theta_1 \quad (50)$$

This integration has also been carried out as discussed in step 7 of Phase L. The results are shown in Figures XVII and XVIII.

Step 4

Integrate this last expression across the radius in order to obtain the radiation emission $(dq)_{w_g}$ which is received by the wall element from the whole gas in the pipe.

$$(dq)_{w_g} = \left[\frac{4}{\pi}(K_eR)\left(\frac{dA_0}{R^2}\right)\right]\int^{\frac{r_1}{R}=1}(E_1R^2)\left(\frac{r_1}{R}\right)\int^{\theta_1=\pi}\left(1-\frac{r_1}{R}\cos\theta_1\right)\left[\frac{1}{a_0^2}\int_0^{\pi/2} e^{-\frac{K_aRa_0}{\cos\psi_0}}\cdot\cos\psi_0\,d\psi_0\right]d\theta_1\,d\left(\frac{r_1}{R}\right) \quad (51)$$

The radiation emission received by the wall element from the gas per unit of wall element area is simply $\frac{(dq)_{w_\epsilon}}{dA_0}$. Since this element is typical of all such wall elements at this cross-section then $\frac{(dq)_{w_\epsilon}}{dA_0} = (q/A)_{w_\epsilon}$.

This result should also be corrected by the real gas transmittance weighting factor $(x)_\epsilon$. This leads to the following corrected result.

$$(q/A)_{w_\epsilon} = \left[\frac{4(\alpha)}{\pi} g(K_e R)\right] \int_{\frac{r_1}{R}=0}^{\frac{r_1}{R}=1} E_1\left(\frac{r_1}{R}\right) \int_{\theta_1=0}^{\theta_1=\pi} \left(1 - \frac{r_1}{R}\cos\theta_1 \left[\frac{1/2}{a_0^2} \int_0^{\pi/2} e^{-\frac{K_a R a_0}{\cos\varphi_0}} \cdot \cos\varphi_0 \, d\varphi_0\right] d\theta_1 \, d\left(\frac{r_1}{R}\right)\right. \tag{52}$$

This integral has not been shown before in this analysis. The results of this integration are shown in Appendix VI.

Step 5

The second component contributing (negatively) to the overall heat flux reaching the wall is the total radiation emission from the wall element

$$(dq)_{w_\epsilon} = -\epsilon_w E_w \, dA_0 \tag{53}$$

Since the walls are assumed to be [illegible], $\epsilon_w = 1$. Also, this wall element is typical of all such elements at this cross-section so

$$(q/A)_{w_\epsilon} = -E_w \tag{54}$$

Step 6

Now combine terms to find the overall net radiation heat flux reaching the wall from the whole gas.

$$(q/A)_{w_R} = (q/A)_{w_G} + (q/A)_{w_\epsilon} \qquad (55)$$

The numerical results of this analysis are shown in Appendix VI.

PHASE E - Total Net Heat Transfer Rate To The Wall

The total net heat transfer to the wall is not the same as the net total heat transfer to the gas control volume located at the wall. This is due to the difference in radiation influences caused by the angles of incidence of radiation beams on the wall area. In addition, a change of sign occurs for the convection component leaving the gas and entering the wall. This causes an additive effect of convection from the local gas ring control volume and radiation from the whole gas.

For the gas next to the wall

$$(q/A)_o = \underbrace{(q/A)_c}_{\text{convection}} + \underbrace{(q/A)_r}_{\text{radiation}} \qquad (56)$$

$(q/A)_c$ is the assumed constant heat transfer rate for the gas next to the wall, $(q/A)_o = -14{,}500$ Btu/hr.ft^2

$(q/A)_r$, at $\left(\frac{r_1}{R} \right) = 1$, has also been calculated and is tabulated in Appendix V, $(q/A)_r = + 2{,}168$ Btu/hr.ft^2 .

Therefore $(q/A)_c$, the convection component, may be determined from equation (56)

$$(q/A)_c = (q/A)_o - (q/A)_r = -16,668 \text{ Btu/hr.ft}^2$$

Now for the wall heat transfer rate,

$$(q/A)_w = (q/A)_{w_c} + (q/A)_{w_R} \qquad (57)$$

But in this case the convection term, although equal in magnitude to the preceding gas convection term, is of opposite sign (going into the wall).

$$(q/A)_{w_c} = -(q/A)_c = +16,668 \text{ Btu/hr.ft}^2$$

The wall radiation term was evaluated in Phase D. Results are shown in Appendix VI. $(q/A)_{w_R} = -1667 \text{ Btu/hr.ft}^2$

Therefore, the total net heat transfer rate to the wall, including radiation for the specified problem is

$$(q/A)_w = 18,335 \text{ Btu/hr.ft}^2$$

PHASE F - COMPARISON OF NUMERICAL RESULTS WITH TWO SIMPLE INDEPENDENT SOLUTIONS

The combination radiation and convection heat transfer problem is sometimes considered, for approximation, as a sum of two independent solutions. Then the temperature distribution is not actually determined. The radiation solution is based on a constant bulk temperature assigned to the whole gas. The convection solution is taken from the results of a pure convection, zero radiation analysis using the appropriate bulk temperature of the gas.

To compare the sum of these independent approximate solutions with the combined solution numerical results, a common bulk temperature of the gas is computed by equation (44) from the combined solution results. $T_b = 1949^{o}R$.

Fluid properties are assumed to have the same value, and all boundary conditions are the same. Therefore, the pure convection coefficients of heat transfer for the gas to the wall is unchanged: $h = 10.0$ Btu/hr.ft^2 ^{o}F from equation (45). Then for the independent pure convection heat flux to the wall, $(q/A)_{w_c} = h(T_b - T_o) = +13,990$ Btu/hr.ft^2 .

The independent constant gas temperature approximate radiation contribution to the wall may be given by: (McAdams (6) , pages 89 and 90)

$$(q/A)_{w_R} = \sigma(\epsilon_g T_g^4 - \alpha_{go} T_o^4) \quad \text{where } T_g = T_b = 1949^{o}R$$

This simple calculation yields $(q/A)_R = +2127$ Btu/hr.ft^2

The sum of these two independent solutions should represent an approximate total net heat transfer rate to the wall.

$$(q/A)_{w} = (q/A)_{w_c} + (q/A)_{w_R} = 16{,}117 \text{ Btu/hr.ft}^2$$

For comparison, the combined analysis equivalent results are given as follows: (for the same gas bulk temperature and the same wall temperature).

$$(q/A)_{\text{convection}} = (q/A)_{w_c} = +16{,}668 \text{ Btu/hr.ft}^2$$

$$(q/A)_{\text{radiation}} = (q/A)_{w_R} = +\ 1{,}667 \text{ Btu/hr.ft}^2$$

$$(q/A)_{\text{Net Total}} = (q/A)_{w} = +18{,}335 \text{ Btu/hr.ft}^2$$

RESULTS

and Discussion of Results

INTRODUCTION

The results of this thesis may be grouped as follows:

(A) A general solution for the gas radiation heat transfer problem for a given gas temperature distribution in a circular pipe.

(B) General numerical results of the preliminary integrations required in the above solution for a range of problem variables.

(C) A procedure for incorporating the radiation solution with the convection heat transfer solution in order to obtain the overall combined heat transfer solution.

(D) A sample numerical solution for the combined radiation and convection heat transfer problem by the methods developed above.

(A) The general solution for the gas radiation heat transfer problem is given in detail in Phase [illegible] and D of the Procedure. The only restrictions imposed by this solution are as follows:

1. The geometry of the gas enclosure is that of a very long circular pipe
2. The radiation properties are constant, so limiting the range of gas temperature extremes in any given problem

3. The pipe wall is assumed to be black.
4. The temperature distribution is specified.

For convenience, but not of necessity, the solution has been written with the additional assumptions that the temperature distribution is axisymmetric, and that the temperature gradient in the axial direction is relatively small.

This solution provides the answer for the net radiation heat transfer rate to any selected control volume element in the gas, (as shown in Phase I of the Procedure) including the effects of all the gas in the pipe and the entire pipe wall area. Also, the net radiation heat transfer to any given wall area element from all the gas in the pipe is given in a similar expression (Phase II of Procedure).

(B) The preliminary integrations required in the above solutions have been carried out numerically for a range of problem variables generally independent of any specified temperature distribution. In particular, the first integration along the length of the pipe has been carried out in each case for a range of (K_aR) values from 0 to $\sqrt{10}$. These results are given in their most useful form in the semi-log plots against (K_aR) in Figures VI, VII, and VIII for the gas control volume cube, and in Figures XIII, XIV, XV and XVI for the wall area element. These results may be used over a wide range of pipe radius values and a wide range of gas absorption coefficients (K_a).

The second integration for each case is carried out in the θ direction around the pipe at constant r. Therefore,

the temperature distribution still remains outside the integral for the axisymmetric case. These integrations are carried out for one sample value of the dimensionless parameter ($K_a P = 0.78$). The results are shown in Figures X and XVIII.

The steps leading up to these second integrations are rather involved due to the requirement of shifting the abscissa of the first integration results to the appropriate ε values. (As described in Phase B of the Procedure). Also, the added complication of dealing with infinite ordinates for several of the curves requires the use of approximate methods of numerical integration to obtain integrals of those curves.

Other integrations in each case across the radius require the introduction of the temperature distribution and therefore can not be given in advance for any general case.

(C) The combined convection and radiation heat transfer solution is developed in general form in Phase A of the Procedure. This solution is limited, as shown in the development, to a special but fairly typical case. In particular, it is voted here that the radiation heat transfer at the wall is assumed to be relatively small compared with the convection component of the overall heat transfer at the wall.

The von Karman constant properties pure convection heat transfer solution [10] was used as a basis for this combined analysis. This solution was chosen as the best analytic solution for the convection problem, which permitted a relatively simple form of integration. The Martinelli solution [5], which covers a wider range of application, is not significantly better for cases (in the region of $N_{Pr} \approx 0.70$),

but requires a much more complex form of integration. The temperature distribution given by the methods of the von Karman solution is not as reliable near the centerline of the pipe, but this weakness was not considered significant enough to require the use of the Martinelli solution as a basis for the combined analysis.

The biggest problem in the combined analysis concerns the evaluation of the $(q/A)_r$ radiation heat flux term as a component of the total heat transfer rate at any point. There appear to be two possible interpretations as to the meaning of the apparent heat flux at any given point. These two concepts are as follows:

(1) The apparent heat flux $(q/A)_{app}$ represents the total heat transfer rate crossing the constant radius boundary of a given ring control volume. That is, it represents the convection heat transfer rate per unit area at that boundary (at r_1), plus the total net contribution of all the radiation heat transfer rate from all inner control volumes (at $r < r_1$) which actually crosses that boundary, per unit boundary area, as radiation heat transfer. This is the most straightforward and physically tangible interpretation, but unfortunately this required radiation function, as specified, is very difficult to obtain. The difficulty is brought about because of the required accounting for the transmittance of the gas and for the adjustment of the contributions of radiation from the gas located upstream and downstream.

(2) The second interpretation defines the apparent heat transfer flux $(q/A)_{app}$ at any point to be a net heat transfer rate for the gas located at that point. That is, it represents the algebraic sum of the convection heat transfer rate per unit area at that point, (at r_1), plus the net radiation heat transfer rate per unit area (normal to the wall) at that same point. This statement is not complete, however, until some depth is assigned to the gas under consideration at this point, since the gas radiation must refer to a volume. This is accomplished by referring the radiation heat transfer, $(q/A)_r$, to a dimensionless unit thickness of constant temperature gas located at this point (at r_1). This is the $(q/A)_r$ term which is used in this development, as defined at the end of Phase B of the Procedure.

It should be noted at this point that the differences in interpretation of the $(q/A)_{app}$ and $(q/A)_r$ terms have no effect on the developed temperature distribution equations (20), (22) and (24), except insofar as what is now meant by $(q/A)_o$ = $(q/A)_{app}$ for the gas at the wall. If $(q/A)_{app}$ is taken in the first sense, then $(q/A)_o = -(q/A)_w$. That is, the total combined heat flux for the gas at the wall is equal in magnitude to the total combined heat flux to the wall (as in the pure convection case). But if $(q/A)_{app}$ is taken in the second sense, then $(q/A)_o \neq -(q/A)_w$. That is, the net combined heat flux for the gas at the wall is not of the same magnitude as the net combined heat flux to the wall. This second case is

the case used in this development. The relationship between $(q/A)_o$ and $(q/A)_w$ for this case is given in Phase A of the Procedure.

(D) In order to demonstrate the application of the combined solution, a numerical sample problem was carried to completion for pure carbon dioxide gas as discussed in Phase C, Phase D and Phase E of the Procedure.

As a starting point, a pure convection solution was carried out to determine an initial trial temperature distribution (Figure XIX), and to establish boundary conditions for the combined convection and radiation problem.

The combined solution then required a reiteration procedure to establish a compatible temperature distribution. This process is described in Phase C of the Procedure, and sample calculations are given in Appendix V. The resulting temperature distribution is given in Figure XX.

The general shape of the combined solution temperature distribution curve is significantly different from the original zero radiation result. The temperature appears to come closer to a uniform value across the pipe, so that the temperature gradient near the wall is relatively greater for a given maximum temperature or bulk temperature of the gas.

The drop off of temperature toward the center of the pipe is of special interest. This is due to the relatively favorable position of this gas for radiation interchange with the rest of the gas in the pipe, coupled with the early assumption that $(q/A)_{app}$ varies linearly with the radius. The integration

procedure used in the von Karman method of analysis, as extended to this application, provide a relatively weak temperature distribution result near the centerline (as exemplified by the zero radiation result). However, it is believed that the drop off in temperature near the centerline does in fact exist, when the axial temperature gradient is nearly uniform with radius.

The temperature calculation right at the centerline does not yield a compatible temperature. This temperature does not significantly influence the other temperature calculations because of the zero area associated with the centerline (r = 0). Therefore, it is considered justifiable to fare in an approximate temperature for this singular point. Other more complex methods of integration, such as used in the Martinelli pure convection analysis, might be used to obtain more accurate temperatures very near the centerline. However, the influence of the gas temperature very near the centerline is very small in the overall heat transfer analysis.

The total net heat transfer rate to the wall was calculated in Phases P and Q of the Procedure. This result showed that the radiation effects led to a gain in the total net heat transfer rate to the wall for the given boundary conditions. The temperature distribution curve, Figure XX, shows that the bulk temperature of the gas is lower for the combined solution example. Therefore, the conclusion is reached that the overall effect of the radiation, compared with a zero-radiation result, is to increase the rate of total

net heat transfer to the wall for a given bulk temperature of hot gas and a specified cold pipe wall temperature.

This conclusion, it must be emphasized, is based on a relatively restricted solution reached with the aid of the limitations and assumptions previously outlined. It is believed that the most serious limitations imposed on this combined heat transfer solution is the restriction to constant fluid properties throughout the analysis. The effect of variable properties may, in fact, be very significant; especially in the boundary layer where the temperature gradient is very steep. It is still possible that the combined effects of radiation coupled with convection, with variable properties considered, might even shift the overall combined analysis to a reduction in the total net heat transfer rate to the wall in some cases instead of the gain shown in this constant properties analysis.

More conclusive results are obtained in comparing this combined analytic solution with the summation of independent convection and simple radiation solutions which do not require complete temperature distribution specification. To accomplish this comparison, the average bulk temperature of the gas was held constant for the two cases as described in Phase I of the Procedure. Then the total net heat flux at the wall was compared for the two methods of solution.

This comparison showed that the total net heat transfer rate to the wall was higher in the combined solution than in the summation of the independent solutions. The radiation component of the heat transfer was lower at the wall in the

combined solution but the convection component of the heat transfer was higher, so that the total net heat transfer rate to the wall was higher for the combined analytic solution.

Therefore, it is concluded that the independent solutions cannot be added together to give an adequate solution for the combination radiation and convection heat transfer problem. The radiation and convection coupling effects must be introduced in a combined analysis to determine the temperature distribution in order to adequately describe the heat transfer solution.

CONCLUSIONS

Thermal radiation exerts a significant influence on the total net heat transfer from a turbulently flowing hot fluid in a circular pipe. A solution to this combined radiation and convection heat transfer problem is presented. Constant fluid properties are assumed throughout the analysis. A sample numerical problem is carried to completion for pure carbon dioxide gas.

The results of this constant properties solution reveal a significant gain in the total net heat transfer to the wall compared to an equivalent zero-radiation solution.

The simple addition of independent radiation and convection solutions based on the gas bulk temperature does not provide an adequate total net heat transfer solution. The coupling effects of radiation and convection must be introduced in a combined analysis to determine the temperature distribution in order to adequately describe the heat transfer solution.

RECOMMENDATIONS

The results of this thesis cannot be considered as a conclusive explanation for the effects of radiation on the general problem of heat transfer from a turbulently flowing fluid in a long circular pipe. Accordingly, the following recommendations are presented as the most promising avenues for significant improvement in the understanding of this subject.

A. Explore the feasibility of using the Martinelli [5] form of analysis for the temperature distribution equations for the combined radiation and convection analysis. The added refinements of a Martinelli type solution would provide better temperature distribution information near the center of the pipe. The effect of this small center section temperature distribution on the overall analysis is relatively small in any case, but it is certainly more significant when radiation is present. Therefore, it is believed that such an improvement should be investigated. The significance of this improvement would be raised for cases in which radiation represents a higher contribution to the overall heat transfer.

B. Extend the combination radiation and convection heat transfer analysis to include the effects of variable properties of the fluid. It is believed that a significant improvement would be obtained by introducing variable properties for the convection properties of the fluid. To include variable

radiation properties of the fluid would require a much more complex radiation solution, but the improvement here would probably not be nearly so significant for most problems.

Therefore, it is recommended that only the properties associated with the convection heat transfer be considered variable. This will lead to a velocity distribution which is a function of the temperature distribution. The effect of this consideration will be especially apparent in the boundary layer region. The overall solution may yield significantly different results - especially for larger temperature extremes.

C. Carry out the combined radiation and convection analysis using the total heat flux concept for $(q/A)_{app}$, the apparent heat flux at any point, instead of the net heat flux concept used in this analysis. The definitions of these two concepts are discussed on page 58 of the results. Because of the more tangible physical picture offered by the total heat flux concept, such a result could be more easily interpreted, and would therefore provide a stronger argument for the solution. The objective of this new more complex analysis would be to provide another numerical solution for a check with the results obtained using the analytically simpler net heat flux concept.

The chief problem which is brought up by this new analysis is the evaluation of the radiation component of the apparent heat flux at any given point. At first thought, one might suspect that the total net radiation flux crossing a given boundary at r_1, is simply the sum of all net radiation

effluxes from all the ring control volumes located at $r < r_1$, summed from $r = 0$ to $r = r_1$. There are two things wrong with this simple summation. First, and most obvious, axial radiation components are not directly accounted for in this summation. Fortunately, this difficulty is not significant for the infinitely long axisymetric pipe, if the axial temperature gradient is assumed to be very small. In that case, mutual symetry exists for all control volumes along the axis at a given radius; such that all the net radiation efflux from a series of control volumes at a given cross-section may be assumed to flow in a radial direction. Therefore, the axial components are accounted for in this indirect manner.

But the second problem is not conquered so easily. This difficulty lies in the coupling effect of radiation and convection. The sum of the total net radiation heat effluxes from all the control volumes inside of the radius r_1 does not necessarily cross the radius r_1 as radiation heat transfer. The mechanism of heat transfer can change to convection heat transfer for a part of that net radiation efflux which is absorbed by the interim gas before reaching the boundary at r_1. Therefore, the sum of all the radiation heat effluxes from all the control volumes inside of the radius r_1, represents the total net radiation flux at the boundary at r_1, plus a part of the convection heat flux at the boundary at r_1. If this sum is then added to the local radial convection heat transfer established by the temperature gradient at r_1, this total will represent more than the actual total heat flux since a part of the convection heat transfer is accounted for twice

A simple solution to this difficulty would be to ignore it. Then the new radiation term $(q/A)_r$ at the boundary r_1 would be simply the following integral.

$$(q/A)_r \simeq \int_0^{r_1} r \left(\frac{dq}{dv}\right)_r dr$$

This would certainly lead to an approximate result, but its significance would be doubtful until it is compared with more exact solutions.

The more exact solution for this expression could be solved by the following procedure:

Carry out a complete three dimensional analysis for each control volume cube at r inside the boundary r_1, but only compute the net radiation which actually penetrates the boundary radius r_1. That is, take into account all one-way wall radiation which reaches the cube, plus all one-way gas radiation from the gas located between r_1 and r_o which reaches the cube, plus the fraction of the total emission from the cube which actually crosses the radius r_1. This last emission term amy require a rather involved calculation to account for the transmittance through some average path length to the boundary at r_1. The net result of these integrations would yield a modified heat flux term for that cube $\left(\frac{dq}{dv}\right)_{r_1}$ for a specified value of the boundary radius r_1. Then this same evaluation should be repeated for other representative cubes inside r_1. Each cube will be representative of all cubes at the same radius at that cross-section, so representing a ring control volume. Next, sum up all control volume contributions

to the flux at the boundary r_1.

Then for this analysis:

$$(q/A)_{r_1} = \int_0^{r_1} r \left(\frac{dq}{dv} \right)_{r_1} dr$$

where the terms inside the integral were evaluated for their contributions at r_1. Therefore, this whole process must be repeated for each selected value of the boundary radius r_1. This would finally lead to a $(q/A)_{r_1}$ distribution with radius which could be used in the total heat transfer rate concept definition of $(q/A)_{app}$.

Then a reiteration procedure would be carried out to establish the compatible temperature distribution. This reiteration procedure would require recalculation of each $(q/A)_{r_1}$ value for each trial because of the variable temperature gas contributions. This process should then converge to the final temperature distribution and total net heat transfer solution.

Needless to say, the work involved for this job would be increased about tenfold over the work required for the solution presented in this thesis (even though constant properties are maintained). However, the radiation solution in this thesis could be used to advantage to reduce this work load, especially for the axial and θ direction integrations.

It would be interesting to see if this solution would yield significantly more accurate results than the other more simple analytic solutions. At least one or more sets of

comparison solutions by different methods will be required in order to show which method is most suitable to provide significant results with a practical amount of work.

A final extension of this radiation term, $(q/A)_{r_1}$, evaluation could be made after the accurate solutions were determined for several cases. Probably a relatively simple approximate two-dimensional breakdown could be developed based on the results of the more accurate three-dimensional solutions. Such a result would be valuable for general engineering application for combination radiation and convection heat transfer problems in long circular pipes.

D. A final recommendation concerns the need for experimental evidence. Accurate experimental evidence is needed concerning the effect of radiation on the overall net heat transfer rate from a hot gas flowing in a cool pipe. In particular, temperature distribution measurements would be most valuable. In this way, a strong check point could be made to compare the analytic development with experimental evidence.

Also, comparative experimental evidence should be obtained to show just how significant the radiation effects become within the range of practical application of fluid flow heat transfer problems where convection usually dominates the heat transfer process. Or from the other extreme, how is a radiation dominated heat transfer problem changed by the coupling effects of added convection heat transfer?

A few careful experiments along these lines should provide guidance to show where it is most profitable to make use of combination convection and radiation heat transfer. This should provide an answer to the choice between a radiating gas and a similar non-radiating gas for a specific heat transfer application.

APPENDIX

APPENDIX I

A. MOMENTUM TRANSFER ANALYSIS [illegible]

Consider control volume as shown in Figures I and II. The flow through this control volume is assumed to be steady axisymmetric turbulent flow. All fluid properties are assumed to be constant. (General references: 3, 4, 7, [illegible], and 9.)

1. Continuity Equation

$$\frac{\partial(\rho v_x)}{\partial x} + \frac{1}{r}\,\frac{\partial(r\rho v_r)}{\partial r} = 0 \qquad \text{(A-1)}$$

2. Momentum Equation - x direction

See Figure IIa

$$-\frac{\partial p}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}\left(\mu r\frac{\partial v_x}{\partial r}\right) = \frac{\rho}{g_0} v_x \frac{\partial v_x}{\partial x} + \frac{\rho}{g_0} v_r \frac{\partial v_x}{\partial r} \qquad \text{(A-2)}$$

(neglecting all second order derivatives in the axial direction as being relatively small)

3. Combine equations (A-1) and (A-2) by addition

$$\frac{\partial p}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(\mu r\frac{\partial v_x}{\partial r}\right) - \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\rho}{g_0} r v_r v_x\right) - \frac{\partial}{\partial x}\left(\frac{\rho}{g_0} v_x^2\right) \qquad \text{(A-3)}$$

4. The last term of this equation represents the axial gradient of the momentum flux in the axial direction. This term is usually small compared with the other terms and may be neglected.

$$\frac{\partial p}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(\mu r\frac{\partial v_x}{\partial r}\right) - \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\rho}{g_0} r v_r v_x\right) \qquad \text{(A-4)}$$

5. Introduce the time average velocity concept for turbulent flow.

$$\left\{ \begin{aligned} v_x &= \overline{v_x} + v'_x \\ v_r &= \overline{v_r} + v'_r = 0 + v'_r \end{aligned} \right\}$$

6. Enter these terms in equation (A-4) and retain only the finite time average terms to obtain the following:

$$\frac{\partial p}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left[r \left(\mu \frac{\partial \overline{v_x}}{\partial r} - \frac{\rho}{g_o} \overline{v'_r \, v'_x} \right) \right] \qquad \text{(A-6)}$$

B. ENERGY EQUATION

Consider the control volume as shown in Figures I and II. (General references: 2 , 4 , 7 , 8 and 9)

1. Apply the first law of thermodynamics

$$dQ - dW_x = di + d\left(\frac{v^2}{2} \right) \qquad \text{(B-1)}$$

$$dQ = + \left[\frac{\partial}{\partial x} \left(K \frac{\partial T}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left(r K \frac{\partial T}{\partial r} \right) + \left(\frac{dq}{dv} \right)_r \right] 2 \pi r \, dr \, dx \, d\tau \qquad \text{(B-2)}$$

where $\left(\frac{dq}{dv} \right)$ is the net radiation heat transfer rate to the control volume per unit volume

$$dW_x = - \frac{1}{r} \frac{\partial}{\partial r} \left[r v_x \left(\mu \frac{\partial v_x}{\partial r} \right) \right] 2 \pi r \, dr \, dx \, d\tau \qquad \text{(B-3)}$$

$$di + d\left(\frac{v^2}{2} \right) = \left\{ \frac{\partial}{\partial x} \left[\rho v_x \left(i + \frac{v^2}{2} \right) \right] + \frac{1}{r} \frac{\partial}{\partial r} \left[r \rho v_r \left(i + \frac{v^2}{2} \right) \right] \right\} 2 \pi r \, dr \, dx \, d\tau \qquad \text{(B-4)}$$

2. Note the continuity equation (A-1) in equation (B-4) and simplify to obtain

$$di + d\left(\frac{v^2}{2}\right) = \left[\rho v_x \frac{\partial i}{\partial x} + \rho v_r \frac{\partial i}{\partial r} + \rho v_x \frac{\partial\left(\frac{v^2}{2}\right)}{\partial x} + \rho v_r \frac{\partial\left(\frac{v^2}{2}\right)}{\partial r}\right] 2\pi r\,dr\,dx\,d\tau \qquad \text{(B-5)}$$

3. Combine equations (B-2), (B-3) and (B-4) and divide by 2πrdrdxdτ.

$$\left[\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) + \left(\frac{dq}{dv}\right)_r\right] + \frac{1}{r}\frac{\partial}{\partial r}\left[r v_x\left(\mu\frac{\partial v_x}{\partial r}\right)\right]$$

$$= \left[\rho v_x \frac{\partial i}{\partial x} + \rho v_r \frac{\partial i}{\partial r} + \rho v_x \frac{\partial\left(\frac{v^2}{2}\right)}{\partial x} + \rho v_r \frac{\partial\left(\frac{v^2}{2}\right)}{\partial r}\right] \qquad \text{(B-6)}$$

4. Combine with the momentum equation (A-[illegible]) and simplify to obtain the following:

$$\left[\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) + \left(\frac{dq}{dt}\right)_r\right] \qquad \text{(B-7)}$$

$$= \rho v_x \frac{\partial i}{\partial x} + \rho v_r \frac{\partial i}{\partial r} - v_x \frac{\partial p}{\partial x} - \mu\left(\frac{dv_x}{dr}\right)^2$$

5. For purposes of this heat transfer analysis, the pressure gradient term and the viscous work term are small compared to the other terms in the quation, so they will be neglected.

$$\rho v_x \frac{\partial i}{\partial x} + \rho v_r \frac{\partial i}{\partial r} = \left[\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r} + \left(\frac{dq}{dv}\right)_{r}\right]\right. \qquad \text{(B-8)}$$

6. Now combine with the continuity equation (A-1), to obtain the following:

$$\frac{\partial(\rho v_x i)}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r i) = \left[\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) + \left(\frac{dq}{dv}\right)_r\right. \qquad \text{(B-9)}$$

7. Assume the temperature gradient in the axial direction is relatively small, so neglect this term.

$$\frac{\partial(\rho v_x i)}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(k\frac{\partial T}{\partial r} - \rho v_r i\right)\right] + \left(\frac{\partial q}{\partial v}\right)_r \tag{B-10}$$

8. Now introduce the time average concept for turbulent flow into equation (B-10) velocity and temperature terms. Retain all finite time average terms.

$$\frac{\partial(\rho \bar{v}_x \bar{i}}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(k\frac{\partial \bar{T}}{\partial r} - \rho c_p \overline{v_r' T'}\right)\right] + \left(\frac{\partial \bar{q}}{\partial v}\right)_r \tag{B-11}$$

It should be noted that in waiting to introduce the time average concept at this point, one relatively important term is not shown in this development. That term is an eddy diffusivity term, $\left[-\left(\frac{\rho}{g_o}\overline{v_r' v_x'}\right)\frac{\partial v_x}{\partial r}\right]$, which might be significant in some analyses. Fortunately, this term is not significant compared to the other remaining terms in the energy equation for purposes of this heat transfer analysis, and its omission is justified. This is due to the high temperature gradients and temperature level encountered in this application.

APPENDIX II

A. RADIATION COEFFICIENTS

(General references: [1], [5] and [6].)

The emission coefficient (K_e) and the absorption coefficient (K_a) are evaluated as functions of the gas emissivity and ($P_g L$) values. The dimension of each of these coefficients is the reciprocal of length. It is assumed in this analysis that these coefficients are equal constants to be evaluated at some given bulk temperature of the gas.

$$K_e = K_a = K \tag{C-1}$$

$$\epsilon_g = (x)_g \left(1 - e^{-KL} \right) \tag{C-2}$$

where L is the mean path length

ϵ_g is the gas emissivity at the gas bulk temperature and average ($P_g L$) value

and $(x)_g$ is the real gas transmittance weighting factor described in step 9 on page 36.

$$(x)_g = \frac{\epsilon_g^2}{2\epsilon_g - \epsilon_{2g}} \tag{C-3}$$

Solving for K

$$K = -\frac{1}{L} \ln \left(1 - \frac{\epsilon_g}{(x)_g} \right) \tag{C-4}$$

This can be simplified to

$$K = \frac{1}{L} \ln \left(\frac{\epsilon_g}{\epsilon_{2g} - \epsilon_g} \right) \tag{C-5}$$

Then solving for the dimensionless parameter, KR

$$KR = \frac{R}{L} \ln \left(\frac{\epsilon_g}{\epsilon_{2g} - \epsilon_g} \right) \qquad \text{(C-6)}$$

B. GEOMETRY OF THE WALL ELEMENT - GAS CUBE RADIATION PROBLEM

See Figure XXI

Note the following dimensions

Angle CDE $= \theta_o$

DE $= r_1$

AB $=$ CD $= R$

AD $=$ BC $= x_o$

CE $= \sqrt{R^2 + r_1^2 - 2\,Rr\cos\theta_o}$

AE $= \sqrt{r_1^2 + x_o^2}$

BE $= z_o = \sqrt{R^2 + r_1^2 - 2\,Rr\cos\theta_o + x_o^2}$

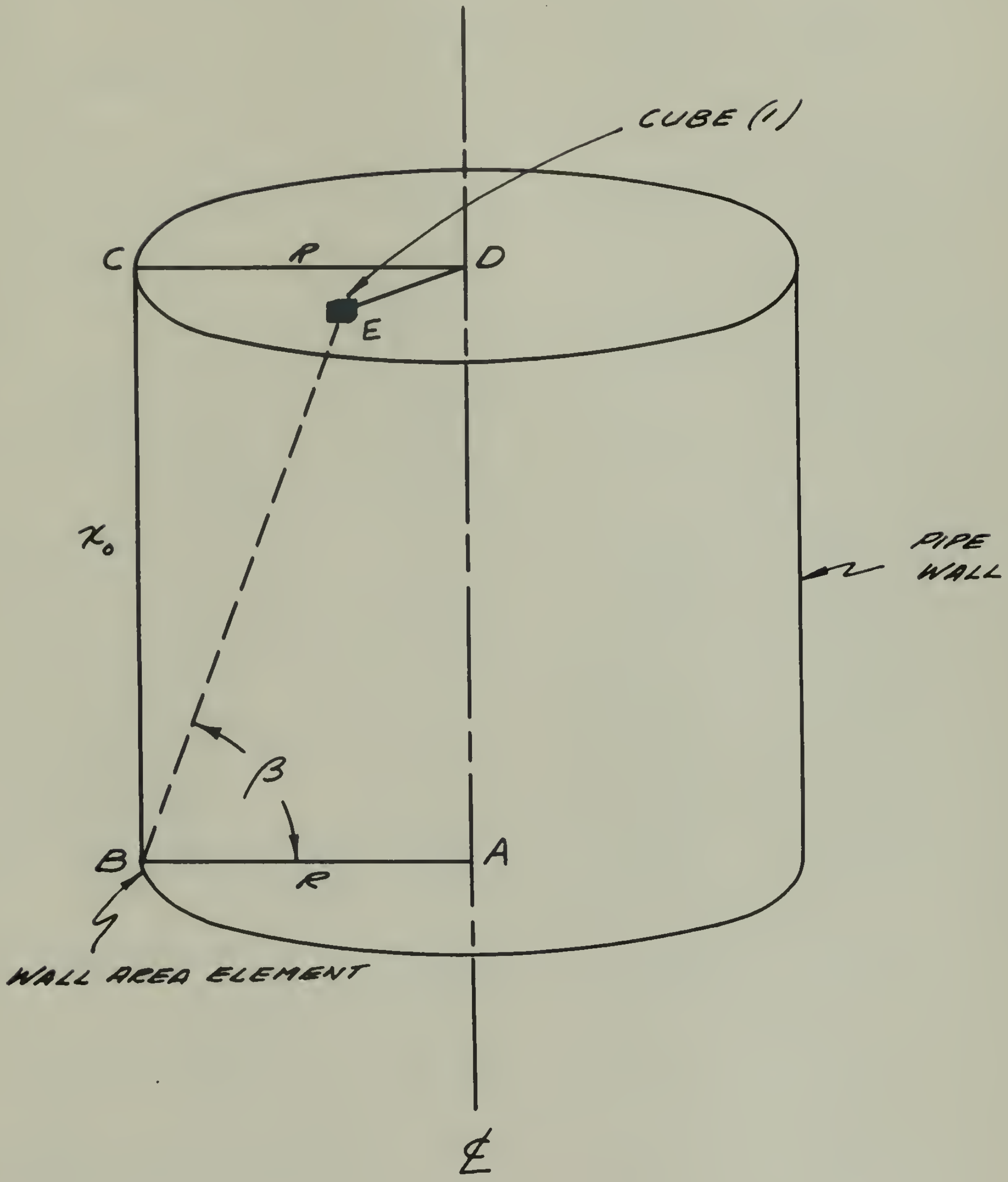
Angle ABE $= \beta$

$$\cos\beta = \cos(ABE) = \frac{(AB)^2 + (BE)^2 - (AE)^2}{2\,(AB)(BE)} \qquad \text{(D-1)}$$

$$\cos\beta = \frac{R^2 + R^2 + r_1^2 - 2Rr_1\cos\theta_o + x_o^2 - r_1^2 - x_o^2}{2R\sqrt{R^2 + r_1^2 - 2Rr\cos\theta_o + x_o^2}} \qquad \text{(D-2)}$$

$$\cos\beta = \frac{R - r_1\cos\theta_o}{\sqrt{R^2 + r_1^2 - 2Rr\cos\theta_o + x_o^2}} \qquad \text{(D-3)}$$

Figure XXI



APPENDIX III

TABULATION OF NUMERICAL RESULTS

A. Convection Analysis

All properties are evaluated at the gas film temperature. (Reference McAdams [6]) for PURE CARBON DIOXIDE GAS

PARAMETER	SYMBOL	VALUE	UNITS
Film temperature	T_f	1275	°R
Gas pressure	P_g	1	Atmosphere
Thermal conductivity	k	0.0304	Btu/hr.ft^2 °F/ft
Specific heat	c_p	0.283	Btu/lb °F
Viscosity	μ	0.0311	Centipoises
Prandtl Number = $c_p\mu/k$	N_{Pr}	0.70	-----
Reynolds Number = $\frac{\rho VD}{g_o\mu}$	N_{Re}	20,000	-----
Pipe diameter	D	0.1667	feet
Gas density	ρ	0.0473	lb/ft^3
Mean gas velocity	V	53.0	ft/sec.
Friction factor	f	0.0065	-----
Wall shear stress	τ_o	0.0134	lb/ft^2
Kinematic viscosity = $\frac{g_o\mu}{\rho}$	ν	1.591	ft^2/hr
Stanton Number = $\frac{h}{c_p\rho V}$	N_{St}	0.00391	-----
Nusselt Number = $\frac{hD}{k}$	N_{Nu}	54.8	-----
Coefficient of heat transfer (zero radiation)	h	10.0	Btu/hr.ft^2 °F
Total net heat transfer flux at wall	$(q/A)_o$	-14,500	Btu/hr.ft^2

PARAMETER	SYMBOL	VALUE	UNITS
Dimensionless coordinate ratio ($y^+/y = \frac{1}{\nu}\sqrt{\frac{\tau_o g_o}{\rho}}$)	y^+/y	6000	1/foot
Dimensionless pipe radius	r^+_o	500	-----
Diffusivity ratio = ϵ_h/ϵ_m (assumed)	α	1	-----
Convection temperature distribution coefficient $f_1(q/A)_o = \frac{(q/A)_o}{\rho c_p \alpha \sqrt{\frac{\tau_o g_o}{\rho}}}$	$f_1(q/A)$	-99.6	°F

F. Radiation Analysis Preliminary Numerical Results

All properties are evaluated at the gas bulk temperature, (Ref. McAdams (3)) for [illegible].

PARAMETER	SYMBOL	VALUE	UNITS
Bulk temperature	T_b	2000	°R
Gas pressure	P_c	1	atmosphere
Pipe diameter	D	0.1667	feet
Average path length = (0.90) D	L	0.150	feet
Average P_cL value	(P_cL)	0.150	ft-atmosphere
Double P_cL value	$2(P_cL)$	0.300	ft-atmosphere
Emissivity at (P_cL)	ϵ_c	0.080	-----
Emissivity at $2(P_cL)$	ϵ_{2c}	0.107	-----
Pipe radius	R	0.0833	feet
Dimensionless emission coefficient	K_eR	0.78	-----
Dimensionless absorption coefficient	K_aR	0.78	-----
Real gas transmittance weighting factor	$(x)_c$	0.1138	-----
Stefan-Boltzmann constant		0.1713×10^{-8}	Btu/hr.ft^2(°R)^4

APPENDIX IV

A. Zero Radiation, Pure Convection Temperature Distribution Calculation

Zone I, ($0 < y^+ < 5$), equation (20), $f_1(q/A)_o = -99.6$

1	2	3	4	5
(y/r_o)	y^+	$(\alpha N_{Pr})y^+$	$(T_o-T) = f_1(q/A)_o \times$ Col. 3	T
0	0	0	0	550°R
	1	0.70	-70	620
	2	1.40	-139	689
	3	2.10	-209	759
	4	2.80	-279	829
0.0038	5	3.50	-349	899

Zone II, $(5 < y^+ < 30)$, equation (22), $f_1(q/A)_o = -99.6$

(y/r_o)	y^+	$5\ell n(1-\alpha N_{Pr}+\alpha N_{Pr}\frac{y^+}{5})$	(T_5-T) = as above	T
0.0038	5	0	0	899
	6	0.653	-65	964
	8	1.753	-175	1074
	10	2.653	-264	1163
	12	3.416	-340	1239
	16	4.661	-464	1363
	20	5.657	-563	1462
	25	6.675	-665	1564
0.05	28.45	7.265	-724	1623
0.0527	30	7.520	-749	1648

Zone III, $(30 < y^+ < r_o^+)$, equation (24) $f_1)q/A)_o = -99.6$

(y/r_o)	y^+	$2.5\ell n(y^+/30)$	$(T_{30}-T)$ = as above	T
0.0527	30	0	0	1648
0.10	56.9	1.605	-160	1808
0.15	85.3	2.610	-260	1908
0.20	113.8	3.331	-332	1980
0.25	142.1	3.890	-387	2035
0.30	170.5	4.342	-432	2080
0.35	199.0	4.729	-471	2119
0.40	228	5.07.	-505	2153
0.45	256	5.359	-534	2182
0.50	284	5.620	-560	2208
0.60	342	6.084	-606	2254
0.70	398	6.464	-644	2292
0.80	455	6.796	-677	2325
0.90	512	7.093	-706	2354
1.00	569	7.357	-733	2381

F. Temperature Distribution Coefficients For Radiation Integral Terms.

The following temperature distribution coefficients for the radiation integral terms in equations (20), (22) and (24) are based on film temperature properties tabulated in Appendix III A.

Radiation temperature distribution coefficients:	SYMBOLS	VALUE	UNITS
Zone I, $(0 < y^+ < 5)$	$\dfrac{\alpha N_{Pr}}{\rho c_p \alpha \sqrt{\dfrac{g_o \tau_o}{\rho}}}$	0.00431	°F/Btu/hr.ft^2
Zone II, $(5 < y^+ < 30)$	$\dfrac{5}{\rho c_p \alpha \sqrt{\dfrac{g_o \tau_o}{\rho}}}$	0.03426	°F/Btu/hr.ft^2
Zone III, $(30 < y^+ < r_o^+)$	$\dfrac{2.5}{\rho c_p \alpha \sqrt{\dfrac{g_o \tau_o}{\rho}}}$	0.01713	°F/Btu/hr.ft^2

APPENDIX V

SAMPLE TEMPERATURE DISTRIBUTION CALCULATION INCLUDING RADIATION

Step 1 - Calculation of E

r_1/R	Assumed T	$(T/100)^4$	$E = \sigma T^4$
0	1950	144,590	24,768
0.10	1991	157,141	26,918
0.20	2019	166,170	28,465
0.30	2030	169,818	29,090
0.40	2030	169,818	29,090
0.50	2020	166,497	28,521
0.60	2001	160,320	27,463
0.70	1967	149,699	25,643
0.80	1912	133,645	22,893
0.90	1798	104,510	17,903
0.95	1651	74,300	12,728
1.00	550	915	$157 = E_W$

Step 2 - Calculation of Radiation Emission from a Gas Cube Located at (r_1/R), $(q/A)_e = -4(x)_g(K_eR)E$,

$$(x)_g = 0.1138 \quad , \quad (K_eR) = 0.78$$

r_1/R	E	$-(q/A)_e$
0	24,768	8794
0.10	26,918	9557
0.20	28,465	10107
0.30	29,090	10329
0.40	29,090	10329
0.50	28,521	10127
0.60	27,463	9751
0.70	25,643	9105
0.80	22,893	8128
0.90	17,903	6357
0.95	12,728	4519
1.00	157	56

Step 3 - Calculation of One Way Radiation From Whole Wall to a Gas Cube Located at (r_1/R).

[From Equation (42)], (See Figure XVIII)

r_1/R	$\int_0^{\pi}(1-\frac{r_1}{R}\cos\theta_0)\left[\frac{L}{a_0^2}\int_0^{\pi/2} e^{-\frac{K_a R a_0}{\cos\psi_0}}\cdot\cos\psi_0\, d\psi\right]d\theta_0$	$(q/A)_w = \frac{4(\alpha)_g E_w(K_a R)}{\pi}\cdot$ (Column 2)
0	1.1330	20
0.1	1.1350	20
0.2	1.1430	20
0.3	1.1675	21
0.4	1.2131	22
0.5	1.2733	23
0.6	1.3355	24
0.7	1.4214	25
0.8	1.5125	27
0.9	1.7999	32
0.95	1.1400	20
1.00	0.5313	9

Step 4 - Calculation of One Way Radiation From Whole Gas to a Gas Cube at (r_1/R).

[From Equation (41)], (See Figure X)

r_1/R	r_2/R	$\int_0^{\pi}\left[\frac{1}{a}\int_0^{\pi/2} e^{-\frac{K_a R a}{\cos\psi}}\, d\psi\right]d\theta_2$	$E_2(\frac{r_2}{R})\cdot$ (Column 3)
0.60	0	3.5340	0
	0.1	3.5955	9,677
	0.2	3.7798	21,520
	0.3	4.1249	35,999
	0.4	4.7994	55,947
	0.5	5.9762	85,224
	0.6	6.40	105,458
	0.7	5.0456	90,369
	0.8	3.3825	61,948
	0.9	2.4312	39,070
	1.0	1.9048	299

Then by numerical integration: $\int_0^1$ [column 4] $d(r_2/R)$ = 51,188

Step 4 - Continued

Tabulation of (q/A) gas to cubes at all radii.

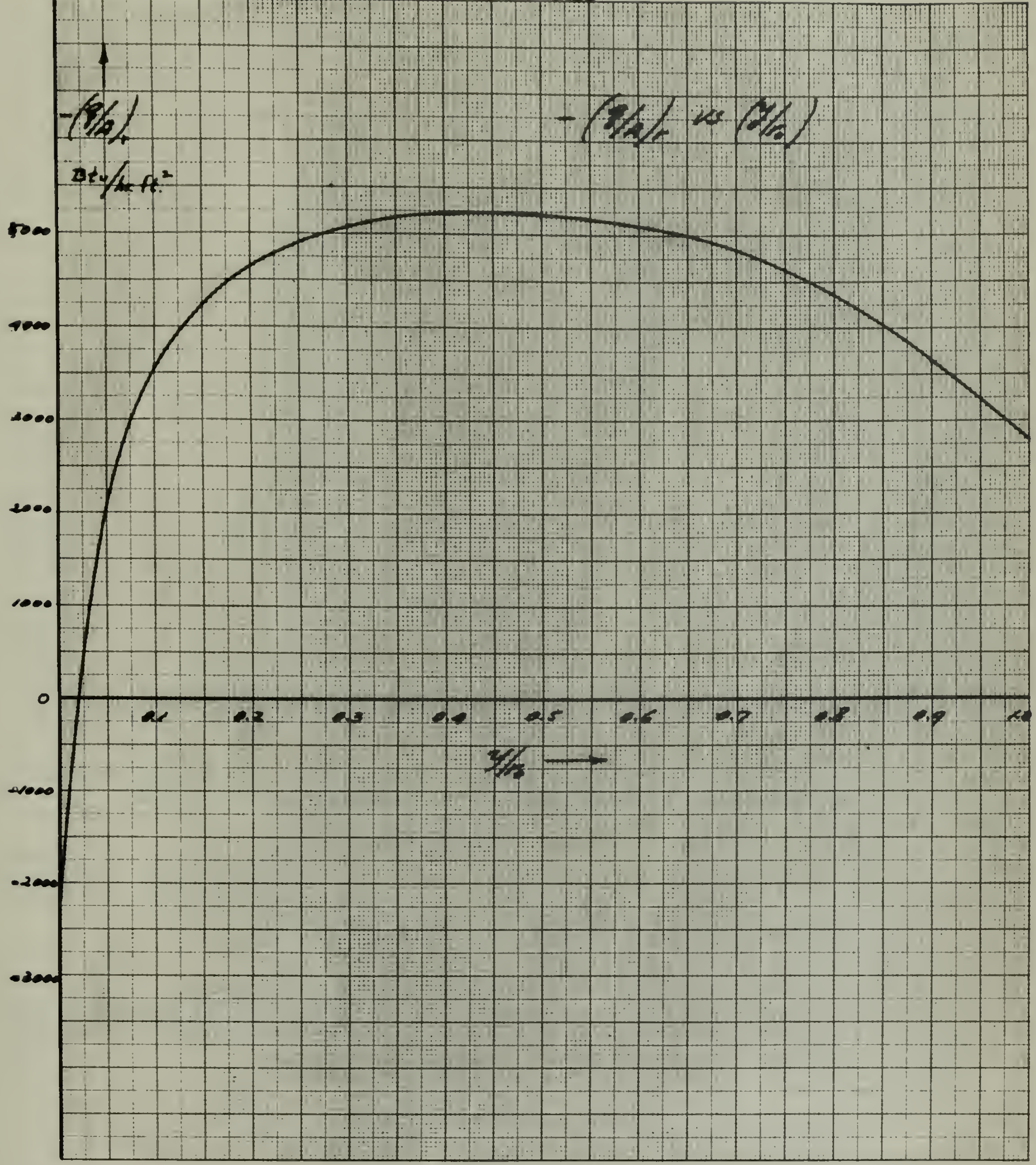
r_1/R	$\int_0^1 E_2\left(\frac{r_2}{R}\right)\left\{\int_0^{\pi}\left[\frac{1}{a}\int_0^{\pi/2} e^{-\frac{K_a R_a}{\cos\psi}}\,d\psi\right]d\theta_2\right\}d\left(\frac{r_2}{R}\right)$	$\left(q/A\right)_{gas} = \frac{4(\alpha)_g(K_eR)(K_aR)}{\pi}\cdot(\text{Col. }2)$
0	67,732	5971
0.1	66,822	5890
0.2	64,709	5704
0.3	62,083	5473
0.4	59,085	5208
0.5	55,691	4909
0.6	51,183	4512
0.7	45,210	3985
0.8	39,362	3470
0.9	31,113	2743
0.95	---	2425 (FARED)
1.0	25,126	2215

Step 5 - Combine Radiation Terms

$$(q/A)_r = (q/A)_{gas} + (q/A)_{emission} + (q/A)_{wall}$$

r_1/R	$(q/A)_{gas}$	$(q/A)_{wall}$	$(q/A)_{emission}$	$(q/A)_r$
0	5971	20	-8794	-2803
0.1	5890	20	-9557	-3647
0.2	5704	20	-10107	-4383
0.3	5473	21	-10329	-4835
0.4	5208	22	-10329	-5099
0.5	4909	23	-10127	-5195
0.6	4512	24	-9751	-5215
0.7	3985	25	-9105	-5095
0.8	3470	27	-8128	-4631
0.9	2743	32	-6357	-3582
0.95	2425	20	-4519	-2074
1.00	2215	9	-56	+2168

Figure XXII


$-(q/A)_r$ vs (y/y_0)
$-(q/A)_r$
Btu/hr ft.^{2}
5000
4000
3000
2000
1000
0
-1000
-2000
-3000
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
y/y_0 →

Step 6 - Plot $(q/A)_r$ versus (y/r_o)

Draw in fare curve (See Figure XXII - small scale of original)

(r_1/R)	(y/r_o)	(y^+)	$(q/A)_r$
1.00	0	0	+2100
	0.0022	1.25	+1925
	0.0044	2.50	+1675
	0.0066	3.75	+1425
	0.0088	5	+1165
	0.0176	10	+250
	0.0264	15	-500
	0.0352	20	-1130
	0.0440	25	-1750
0.95	0.05	28.45	-2074
	0.0527	30	-2210
0.90	0.10	56.9	-3582
	0.15		-4210
0.80	0.20		-4631
	0.25		-4910
0.70	0.30		-5095
0.60	0.40		-5215
0.50	0.50		-5195
0.40	0.60		-5099
0.30	0.70		-4835
0.20	0.80		-4383
0.10	0.90		-3647
0	1.00	569	-2803

Steps 7 and 8 - Obtain the $(q/A)_r$ integrals in the temperature distribution equations (20), (22) and (24).

Zone I, $(0 < y^+ < 5)$

$$\int_0^5 (q/A)_r \, dy^+ = +8,377 \text{ at } y^+ = 5 \text{ by Simpson's Rule}$$

Zone II, $(5 < y^+ < 30)$, integration by planimeter

y^+	$\dfrac{(q/A)_r}{\left[\dfrac{5}{\alpha N_{Pr}} - 5 + y^+\right]}$	$\displaystyle\int_5^{y^+} \dfrac{(q/A)_r \, dy^+}{\left[\dfrac{5}{\alpha N_{Pr}} - 5 + y^+\right]}$
5	+165.90	0
10	+23.06	+422
15	-29.17	---
20	-53.29	+176
25	-64.47	---
28.45	-67.79	-362
30	-68.76	-466

Steps 7 and 8 - Continued

Zone III, $(30 < y^+ < r_o^+)$, integration by planimeter

(y/r_o)	$\dfrac{(q/A)_r}{\left[y^+(1 - y^+/r_o^+)\right]}$	$\displaystyle\int_{30}^{y^+} \frac{(q/A)_r\,dy^+}{\left[y^+(1 - y^+/r_o^+)\right]}$
0.0527	-77.76	0
0.10	-69.05	-2006
0.15	-58.05	-3827
0.20	-50.87	-5377
0.25	-46.04	-6745
0.30	-42.69	-7990
0.40	-37.19	-10270
0.50	-36.52	-12376
0.60	-37.34	-14467
0.70	-40.51	-16672
0.80	-48.14	-19175
0.90	-71.22	-22447
1.00	$-\infty$	------

Steps 9 and 10 - Calculation of New Temperature Distribution

By Equations (20), (22) and (24)

y^+/r_o ①	y^+ ②	Radiation Integral (Step 8) ③	Radiation Coefficient (App. IV) ④	③ × ④ ⑤	$(q/A)_o$ Term (App4) ⑥	⑥-⑤ ⑦	T (°R) ⑧
0	0	0	0	0	0	0	550
0.0088	5	+8377	0.00481	+40	-349	-389	939
0.0176	10	+422	0.03436	+14	-264	-278	1217
0.0352	20	+176	↓	+6	-563	-569	1508
0.05	28.45	-362	↓	-12	-724	-712	1651
0.0527	30	-466	↓	-16	-749	-733	1673
0.10	56.9	-2006	0.01718	-34	-160	-126	1798
0.15		-3827	↓	-66	-260	-194	1866
0.20		-5377	↓	-92	-332	-240	1912
0.25		-6743	↓	-116	-387	-271	1945
0.30		-7980	↓	-137	-432	-295	1967
0.40		-10270	↓	-176	-505	-329	2001
0.50		-12376	↓	-213	-560	-347	2019
0.60		-14467	↓	-249	-606	-357	2029
0.70		-16672	↓	-286	-644	-358	2030
0.80		-19175	↓	-329	-677	-348	2020
0.90		-22447	↓	-386	-706	-320	1992
1.00	569	------	↓	----	-735	----	1950

(FAIRED)

APPENDIX VI

CALCULATION OF RADIATION TO WALL

r_1/R	T_1	E_1	$E_1\left(\frac{r_1}{R}\right)\int_0^{\pi}\left(1-\frac{r_1}{R}\cos\theta_1\right)\left[\frac{1}{a_0^2}\int_0^{\pi/2} e^{-\frac{K_a R a_0}{\cos\varphi_0}}\cos\varphi_0\, d\varphi_0\right] d\theta_1$
0	1950	24768	0
0.1	1992	26973	3052
0.2	2020	28521	6520
0.3	2030	29090	10184
0.4	2029	29032	14086
0.5	2018	28465	18118
0.6	2001	27463	21981
0.7	1967	25643	25507
0.8	1912	22893	27710
0.9	1798	17903	29003
1.0	550	157	83

The integral of column 4 with respect to r_1/R from 0 to 1 is 16,138 by Simpson's Rule

$$(q/A)_{w_g} = \frac{4(x)_g (K_a R)}{\pi} \times (16{,}138) = 1{,}824 \text{ Btu/hr.ft}^2$$

$$(q/A)_{w_\epsilon} = - E_w = -157 \text{ Btu/hr.ft}^2$$

$$(q/A)_{w_R} = (q/A)_{w_g} + (q/A)_{w_\epsilon}$$

$$(q/A)_{w_R} = 1667 \text{ Btu/hr.ft}^2$$

Net radiation to wall from the whole gas

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